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ABSTRACT

High school mathematics (beginning with algebra) is widely regarded as the "gatekeeper" to college. It is also the subject students in U.S. public schools fail most often. As the standards movement gains momentum, students who are members of subordinated populations continue to perform worse on standardized measures of mathematical skill than do their mainstream peers. Fundamentally linked to these problems is the nature of mathematics education and the popular notion of "mathematics" as a discipline. Mathematics education research has never viewed equity as a central concern, the "objective" nature of mathematics in general has never called for examination of the complex political, economic, and ideological forces that shape the terrain of mathematics education in public schools. This study uses a variety of statistical and ethnographic techniques to illustrate ways in which traditional high school mathematics education supports hegemony and maintenance of the status quo. The study uses multiple regressions, analysis of variance, and path analysis to investigate students' attitudes toward mathematics and to explore factors contributing to success in mathematics. The more math students take, the more they resist the subject and the less it appears useful in everyday life. The study relies upon a critical awareness of the social contexts of both mathematics and mathematics education research to contrast "radical" mathematics education with "traditional" mathematics education. Focus group interviews with students, interviews with teachers, and transcript analysis suggest that teacher expectations of student ability have a strong effect on students' mathematical confidence. Confidence, in turn, is a key determinant of students' success or failure in school mathematics. Teachers tend to equate mathematics "ability" with the display of appropriate behaviors. Tracking students into college-prep and non-college-prep courses of study continues to fundamentally limit student opportunity. Finally, a case study illustrates the challenges of implementing policy measures grounded in a critical mathematics education paradigm. Math teachers must always consider the larger societal context in which they practice mathematics education if they are to understand the problematic role that subject plays in the education of students, especially those who are members of subordinated groups. (Contains 253 references.) (Author/ASK)

CENTRAL TENSIONS
A Critical Framework for Examining High School Mathematics and
Mathematics Education

by

Katrine Gram Jacobson

Claremont Graduate University and San Diego State University 1999

High school mathematics (beginning with algebra) is widely regarded as the "gatekeeper" to college. It is also the subject students in U.S. public schools fail most often. As the standards movement gains momentum, students who are members of subordinated populations continue to perform worse on standardized measures of mathematical skill than do their mainstream peers. Fundamentally linked to these problems is the nature of mathematics education and the popular notion of "mathematics" as a discipline. Mathematics education research has never viewed equity as a central concern, the "objective" nature of mathematics in general has never called for examination of the complex political, economic, and ideological forces that shape the terrain of mathematics education in public schools.

This study uses a variety of statistical and ethnographic techniques to illustrate ways in which traditional high school mathematics education supports hegemony and maintenance of the status quo. The study uses multiple regressions, analysis of variance, and path analysis to investigate students' attitudes toward mathematics and to explore factors contributing to success in mathematics. The more math students take, the more they resist the subject and the less it appears useful in everyday life.

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Finally, a case study illustrates the challenges of implementing policy measures grounded in a critical mathematics education paradigm. Math teachers must always consider the larger societal context in which they practice mathematics education if they are to understand the problematic role that subject plays in the education of students, especially those who are members of subordinated groups.

Dedication

To John,
without whom this paper would not exist,

and

To Hanne and Eric Jacobson,
without whom I would not exist.

You are remarkable people I'm proud to call my parents

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Introduction

Although public education in general has come under fire from many directions, one academic subject is often sighted at the intersection of the crosshairs. Many see mathematics slipping below the surface of the quagmire public schools have become. Mathematics education in the United States is compared to that taught in other nations: Japan and Germany, most notably, in the TIMSS Report (1995). As the media has taught people to expect, U.S. students score "at the bottom" of the list when compared to their international peers. President Clinton's Education 2000 Agenda (taken from Bush's GOALS 2000 vision) called for the nation to become number one in the world in mathematics. Nobody took time to even envision the kind of mathematics which would be "mastered," even fewer paused to consider the possible costs of such a pursuit.

Mathematics in today's public high schools, though viewed as pre-eminent among traditional academic subjects, is resisted by students. Mathematics is the subject most students fail. Girls in the United States are allowed, even encouraged, to become poor math students (as evidenced by the talking Barbie who whined, "I hate math!") while the SAT and other college admission criteria rely heavily on assessment of traditional mathematical skills and knowledge. Students must learn to graph abstract quadratic equations and they are told that yes, sometime, they really will use factoring "in real life." Teachers, parents, and counselors tell students they must continue formal study of mathematics because "it will be important later." The practical value of school mathematics differs little from the unknown value of taking mineral supplements, people

better take it because it just might help. Too much will kill you. Just ask the eighteen-year-old who has managed to survive all three years of the mathematics required for application to the University of California, where he or she plans to major in journalism... or the high school dropout who works for minimum wage at Taco Bell giving change from a register that indicates the appropriate number of nickels, dimes, and quarters on a screen attached to a computer.

High school mathematics is closely tied to traditional views of mathematics and research in mathematics education. While other subjects (history and literature, for example) have been deconstructed and re-examined from various points of view, mathematics has retained a revered distance from criticism and serious revision. "Mathematics reform" is a product of legislative action that is informed by scholars at major research universities who got there because they were able to "play the game" where success in school mathematics is an important factor; in no way has the nature of high school mathematics been seriously altered over the course of the past 50 years. In fact, "mathematics reform" is a term that has come to be associated with "math wars" and swirling controversy that only superficially even begins to scratch the stained glass windows housing the holy relic itself. Interestingly, the media's coverage of the "controversy over mathematics education" has led the public to believe that change, in fact, had occurred. People who question the value of high school mathematics are perceived as ignorant. Those who "know math" want to save "real math" and not experiment with "new math" (which is new only in the sense that factoring and quadratic functions appear clothed in different outfits).

While any serious criticism of issues embedded in high school mathematics is thus dispersed into wayward dust storms, thousands of young people each year are enrolled in mathematics courses that look, by and large, the same as they did in 1950. Some succeed. Most do not. Poor students, students of color, and girls do worse than members of other groups. The University of California, thanks to Proposition 209, must rely on SAT scores (and their highly-valued "quantitative reasoning" section) for a large part of admission criteria. UCSD admitted 45% fewer Latino students in 1998; U.C. Berkeley reported a 65% drop in "minority" admissions this year. The 1996 CPEC High School Eligibility study reports that the number one reason why students fail to qualify for application to the University of California is inadequate mathematics course completion. SAT scores and family income have a nearly one-to-one correlation. Just who is being served by high school mathematics today? The same people who benefited in 1950?

People doing educational research, indeed anybody interested in the future of young people educated in a country purported to be a democracy, must ask new questions about the nature of mathematics and mathematics education. If not for the incredible betrayal and tragedy implicit in the numbers above, today's situation might remind one of Samuel Becket's play Waiting for Godot. From a distance, and from a classical perspective, today's mathematics education "debate" is typical of absurdist theatre. The world is watching as the main characters wait for someone that never arrives.

"Just group the like terms in this polynomial."

"Teacher, when am I gonna use this stuff?"

"Engineers do this. Now get to work."

How many high school graduates become engineers, anyway? Do *engineers* ever have to group like terms in a polynomial expression?

The cost of nostalgically and closemindedly viewing high school mathematics through a 2000-year-old European-style monocle is tremendous. It is time to open another eye and view the horizon that stretches behind this thing we revere as "high school mathematics."

Underlying Premises

Any discussion about the nature of high school mathematics and its consequences for students and the educational system must address two important subjects. **The first premise of this study is that two factors share a particular relationship with each other.** First, the nature of “knowledge” and the epistemology of “mathematics” must be examined. Just what, exactly, is this thing called “mathematics?” This first consideration seems reasonable: how can one expect to teach or learn something unless those who learn take the subject to be the same as those who teach? Second, one must consider the nature of the “mathematics education community,” “mathematics education research” and “mathematics reform.” Clearly, educational policy at the state and national level has strong ties to research universities and special interest groups. Who drives these movements? What guiding questions are used to direct research and other (funded) attempts to collect, analyze, and evaluate data? It would be interesting to examine particular relationships between the first factor (epistemology of mathematics and knowledge) and the second factor (mathematics education/research).

Educational researchers are expected to conduct study in areas where “problems” have been documented. Mathematics, evidently, is a subject plagued by problems. Once problems have been identified, one is instructed to develop research questions and a methodology that will support inquiry into the problem. It is here that critique begins to unravel, because the questions one asks and the methods one chooses are grounded in one’s fundamental beliefs about the problem to be studied. To assume that one’s research will be “objective” and “bias-free” is absurd. However, once that has been established,

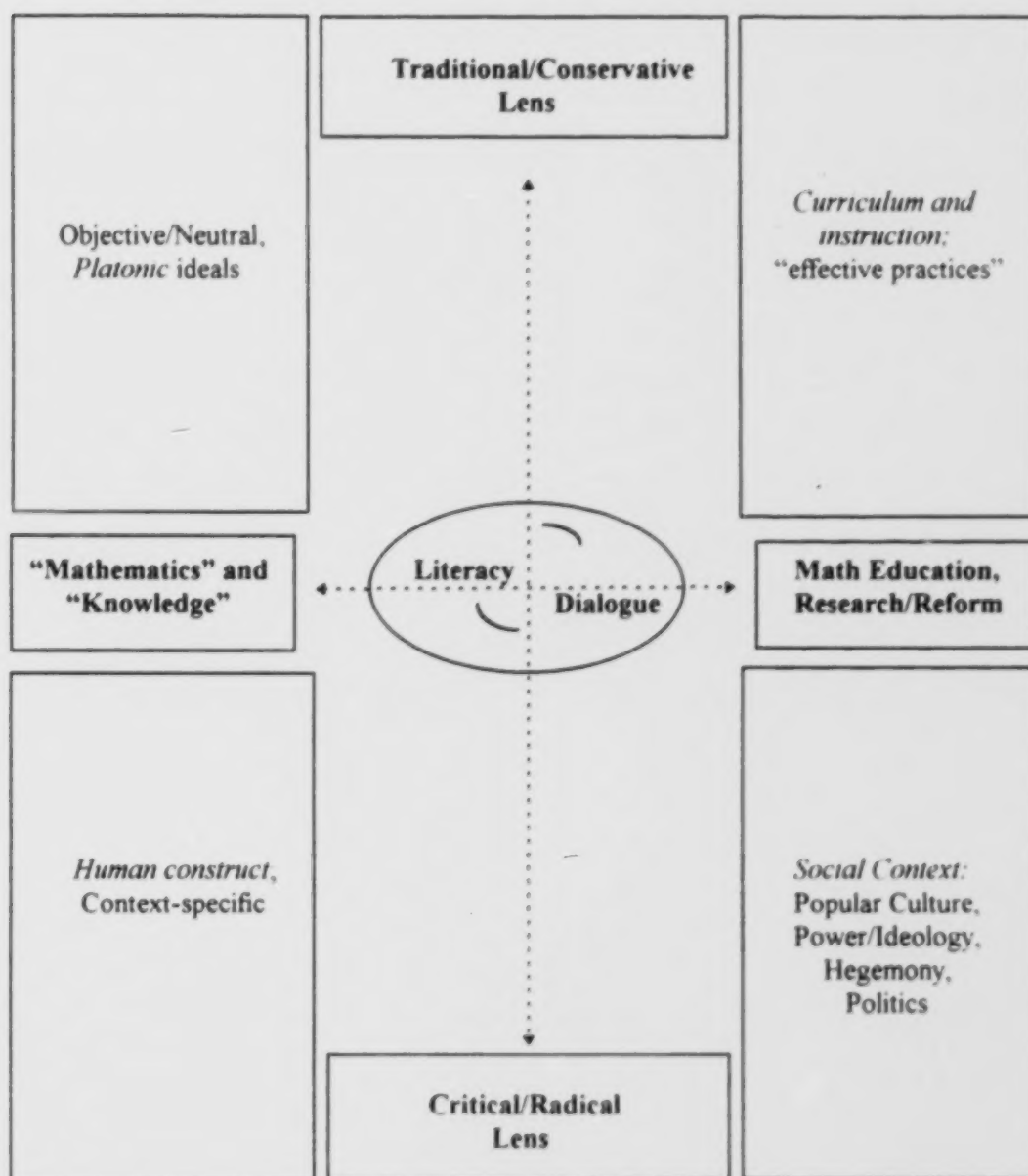
one might accept the premise that one's view of the problem (indeed of the world in which that problem is lodged) depends on the lens through which one views everything. Once one accepts this premise, it is easy to see why mathematics education has changed so little over the course of the past 50 years.

If constructive inquiry into the nature of "high school mathematics" depends on examination of the relationships between the epistemology of mathematics/knowledge and the mathematics education community/research, then it also results from viewing "the problem" through an alternative lens. **Therefore, the second major premise of this study is that research questions dealing with "high school mathematics" lie on a continuum between two extremes: conservative/traditional and critical/radical.** As the names would imply, "conservative/traditional" approaches are grounded in the belief that one's research must support the status quo or ideals from past eras. "Critical/radical" approaches are less easily characterized, but they are grounded in the belief that scholarship must advance change. Very few efforts to examine "high school mathematics" are framed by a "critical/radical" lens. This, perhaps, is why mathematics education has changed so little over the course of this century.

If one accepts these two key premises, definition of the "problem" of "high school mathematics" and the accompanying research questions are informed by the researcher's position on the plane defined by these two axes. Figure 1 describes this model. I contend that the only way to navigate the treacherous journey between "mathematics/knowledge" and "mathematics education/research" is also the only way to negotiate between traditional/conservative and critical/radical lenses **dialogue that**

begins with new questions Current research in mathematics education lacks both dialogue and new questions. Important voices are (unnoticed and) silent. It is therefore essential to explore all quadrants before recommendations or conclusions may be reached. This is an a-traditional and eclectic mission whereby methodology must address questions normally associated with different approaches. Essentially, this study attempts to reveal the many ways in which research dealing with "high school mathematics" has failed to break new ground. It is an effort to begin authentic dialogue. It is my hope that this dialogue may spur change that benefits those who have been most underserved by the system in which "high school mathematics" has been taken for granted to be a subject of undisputed value and merit.

Figure 1
Central Tensions: Mathematics and Math Education



Chapter One: Statement of the Problem

"Knowledge of mathematics is essential for all members of society. In order to participate fully in democratic processes and to be unrestricted in career choice and advancement, individuals must be able to understand and apply mathematical ideas" (Stanic & Hart, 1995, p. 258). Most educational researchers express the view that knowledge of mathematics is important for many reasons. If so, students in the United States fail to demonstrate mastery of certain mathematical concepts. Recent statistics strike fear into the hearts of those who see a strong link between success in school mathematics and success in life.

The 1992 NAEP Report (Educational Testing Service, 1992) states that across grades 4, 8 and 12, 25% or fewer students were estimated to be at the "proficient" level or beyond, where students should exhibit evidence of solid academic performance in school mathematics (p. 3). Only 2-4% of students functioned at the "advanced" level. Asian/Pacific Islander and White students showed the highest proficiency, African-Americans showed the lowest. At grade 12, males outscored females. Fewer than half of Latino, African-American, and Native American students scored at the "basic" level. Students in Iowa, North Dakota, Minnesota, Maine, New Hampshire, and Wisconsin scored highest of all eighth-graders. Girls lag behind boys in terms of enrollment in calculus and physics, both high-level mathematics courses (*NCTM Bulletin*, 1995).

Trends are equally disturbing. The average performance for Whites increased at all three grades, whereas African-American students were the only other group to show

gains and these gains occurred only in terms of the number of students who scored at or above the "basic" level (Secada, 1992). Too few students are electing to take advanced mathematics courses, and only 5 out of every 100 students completes four years of college-prep mathematics. Fewer than 50% of students in urban schools take any math beyond one year of algebra; one-fifth take no algebra at all (ETS, 1992).

The NAEP report suggests a positive relationship between the number of college-prep math courses completed by a student and his/her math achievement test scores; this means that problems are also significant at the college level. According to the 1992 NAEP, only 6% of students in grade 12 demonstrated consistent success in the areas of geometric relationships, algebra and functions. Since these skills are prerequisites for success in any college-level mathematics course, one may conclude that only a small fraction of graduating students are mathematically prepared to succeed in college-level mathematics. Sheila Tobias (1990) observed that first-year experiences in courses make a big difference in students' choices of majors; poor experiences early in college could radically alter a student's plans. In 1989, African-Americans comprised 12% of the national population and earned 1% of all Ph.D.s in mathematics and science. Latinos, though 9% of the national population, earned 2% of math/science Ph.D.s. White women, comprising 43% of the national population, earned only 13% of the Ph.D.s in math and science (NSF, 1989).

Statistics at the college level indicate that fewer students in general are entering and graduating from science, mathematics and engineering majors at colleges and universities. The situation is particularly acute for women and traditionally disadvantaged

students. Green (1989) notes a decline in all undergraduate freshmen planning to major in science in the last 20 years (from 11.5% to 5.8%), as well as a major decline in the number of college students preparing for careers in science or math teaching. In the last 20 years, the number of women enrolled in science, mathematics, and engineering (SME) majors has dropped from 8.8% to 5.1% (Green, 1989; NSF, 1989). According to the National Science Foundation (1990), more than 50% of students who enter science, mathematics, or engineering majors as freshmen eventually switch to other majors. African-American, Latino, and Native American students have especially low retention in SME majors (NSF, 1989).

Another dimension of the problem of high school mathematics education relates to its consequences for future endeavors. An ETS Study (Chronicle of Higher Education, 1995) states that even among high-ability 1980 High School senior less than half (49%) got college degrees within seven years of graduation. The study also asserts that student success depends, to a large degree, on the quality of secondary mathematics programs. In mathematics, the influence of participating in K-12 programs seems more pronounced (p. A25). Elaine Seymour (1992) warns that unless current trends are reversed, the nation will have difficulty maintaining standards of science and math teaching in high schools (p. 230) and she cites a report by Mullins and Jenkins (1988) predicting that American competitiveness in science- and technology-dependent sectors of the world economy will be undermined unless the United States is able to "produce" more mathematically-adept students.

These statistics, however, are symptomatic of an underlying problem. Sal Restivo (1993) articulates this well.

If pure mathematicians have to rely on their own cultural resources, their capacity for generating innovative, creative problems and solutions will progressively deteriorate. As a consequence, the results of pure mathematical work will become less and less applicable to problems in other social worlds. (p. 255)

Everybody Counts, a report by the National Research Council (1989), warned that "innumeracy threatens democracy because people who understand mathematics will join a technologically powerful elite" (p. 26). However, the report also stated that "learning mathematics requires special ability, which most students do not have" (p. 10).

Apparently, mathematical knowledge is important, but only a very few should hope to acquire it. Elsewhere in *Everybody Counts*, the NRC writes that "mathematics holds the key in an information based society" and that "unless corrected, innumeracy and illiteracy will drive American apart" (p. 14). To what degree do our schools expect every student to learn mathematics? Is it really considered imperative for all students?

Although many critics of high school mathematics education claim that all students are suffering from substandard treatment, few elaborate on the ways in which mathematics differentially affects students. "If there is a general crisis in mathematics education in the nation, then the situation for those least well-served by the current system is even more desperate" (Secada, 1995, p. 16) and "in schools serving poor communities, teachers and students alike have been trapped for too long in a web of poor preparation, low

expectations, and limited resources" (p. 20). Carey, Fennema, Carpenter and Franke (1995) cite another NAEP Report (1988) when describing "major inequities in mathematical outcomes for women, certain ethnic groups, ESL students and poor kids" (1993). They comment that people from these groups tend not to participate in math-related occupations.

A critical factor in the problem of high school mathematics is tracking. According to the NAEP (1992), over two-thirds of schools engage in tracking, placing eighth graders in different math classes according to ability. Tracking systems provide students from subordinated populations with few opportunities to learn higher-level mathematics (Oakes, 1990). Tracking systems are also characterized by inequitable distribution of resources (Tate, 1995). A student's primary language contributes to his or her ability to receive effective mathematical instruction, since a higher-level math curriculum is sometimes denied to language-minority students (Oakes, 1990). Despite these and other powerful arguments against the practice, tracking is virtually unchallenged in schools which boast the highest scores on standardized tests. Why should students' scores on multiple-choice, norm-referenced exams allow schools to perpetuate overt segregation of students in mathematics and other subjects? The reputation of a school is linked to its students' performance on measures of mathematical aptitude. That such a construct as "mathematical aptitude" has been *allowed* to exist (given all the intervening factors), not to mention *supported* by educational research, is incredible at best.

An important dimension of the problem, then, is the way in which equity has failed to appear as a major strand in mathematics education research. Walter Secada (1992)

describes what he calls equity's "marginal status relative to mainstream mathematics research (p. 654) and Carey et. al. (1995) write that "it is clearly documented that inequities exist and that they can be alleviated. However, in most classrooms and to many math educators, equity is seldom a concern" (p. 93). Students from low socio-economic backgrounds "receive impoverished or low-level remedial instruction, often in the form of recitation teaching" (Rueda, 1992, p. 1). Especially in today's standards-driven era, inclusion of equity concerns into mathematics education research is critical. It is disturbing how little research calls into question the naively ignorant rallying cry that "all students must achieve the same high standards" when school funding varies so widely across the nation.

A final aspect of the problem of high school mathematics relates to the inattention that has been paid to dialogue and communication in mathematical contexts. Secada (1995) illustrates the crucial role of communication and dialogue within a classroom, particularly where mathematics is taught:

Instructional practices in schools are seldom designed to embrace the strengths that culturally diverse children bring with them to school, nor do they regularly provide bridges that would enable children from distinct subcultures to develop fluency in both their own and mainstream cultures. (p. 23)

Goldenberg (1991) describes the common perception that students with limited English proficiency fundamentally require drill, review and redundancy in order to progress academically (citing Brophy and Good, 1986), and Kang & Pham (1995) warn that "focus

on symbol manipulation may make some language minority students with less developed English proficiency feel comfortable, but it is not helpful in the long run as more is required than symbol manipulation, particularly at higher levels" (p. 27). Although "current trends and standards place increasing emphasis on communication in mathematics instruction and assessment" (Brenner, 1994), little attention has been spent on studying ways to develop cross-cultural communication in mathematics classrooms. Little, if any, research in mathematics education is focused on the ways in which the "teaching and learning [of mathematics] consist of an interaction between persons for the purpose of developing and sharing meanings" (Khisty, 1995, p. 279).

In summary, the problem of "high school mathematics" is complex and multidimensional. Perhaps the most important aspect of the problem is described by Peter Appelbaum (1995), who writes, "Defending what is there...often becomes more important than transforming curricula or expanding one's educational horizons" (p. 344). Elaine Seymour (1992) comments that "a far greater proportion of the problems described by [people who switch out of science or math majors] arises from structural and cultural sources than from problems of personal inadequacy" (p. 234). Marilyn Frankenstein (1990) echoes this concern:

The statistical picture of race and gender inequalities emphasizes that, most often, personal situations are not unique and not the result of individual failure, rather, they are due to the failure of our society to ensure equality and justice for all (p. 343)

Seymour (1992) suggests that research on mathematics education has tended to focus attention on peripheral issues and to divert it away from an honest examination of institutional problems. Perhaps it would be wise to look beyond the institution and its environment into the discipline of mathematics itself. What exactly is this dragon we send all students to slay?

Review of relevant literature must seek to address each of these issues.

Examination of this literature and current educational research suggests that policies related to mathematics education are grounded in very traditional perspectives. Only a few researchers appear concerned about the critical implications of mathematics education, even fewer provide theoretical frameworks that teachers, students, or parents might use when working toward development of critical mathematics education. These few deserve attention, what they have to say must not be drowned within the sea of mainstream math education research.

Chapter Two: Literature Review

Recent scholarship in the field of mathematics education tends to focus on cognitive approaches to the teaching of "mathematics," a set of truths teachers must help students to apprehend. In this sense, mathematics education research has changed very little in the past 50 years. Equity occupies a marginal position in the mathematics education picture. Similarly, the definition of the "mathematics" students must know and be able to do has remained fairly constant, despite the vast numbers of students who continue to "fail" at this pursuit. It is imperative, therefore, to examine the contributions of the very few researchers who critically redefine both mathematics education research and the discipline of mathematics itself.

Mathematics Education Research

Traditional mathematics education research has tended to focus on classroom activities, optimal sequence of topics or individual cognitive development (Appelbaum, 1995). Leder (1995) identifies two broad approaches that dominate current research on formal mathematics learning: research on classrooms (groups of students) and research on students' ideas and intuitive learning strategies (individual students). Most school-based conceptions of equity (Secada, 1994) focus on how teachers work with differences among students such as race, gender, ethnicity and class. Differences manifest themselves in the classroom as differences in reasoning (Tharp, 1995, p. 3). The nature and scope of scholarship has been limited to development and testing of new methods and materials

(Kilpatrick, 1992). Davis (1993) observes that any changes in the mathematics curriculum center around specific mathematical topics and instructional approaches; few researchers question mathematical structures or means in their own terms (p. 189).

However, Michelle Fine (cited in Apple, 1995) writes that "focusing only on any one element misses the depth of the problem and what may be necessary for lasting transformations" (p. 344). Secada (1995) echoes this concern:

Notions of access to quality instruction and of the greater texture of possibilities at the intersection of cooperative groups and cross-cultural communication, and even of how diversity itself is thought of, become constrained if all that can be referred to is a single dimension - for example, cross-cultural communication. (p. 154)

Traditional research in mathematics education fails to interrogate the nature of mathematics itself. Jung (1993) observes that "the underlying conceptual framework is not often described explicitly in papers. It is the empirical studies that dominate" research in mathematics education (p. 135). In traditional research, mathematics has an assumed status as an objective, absolute body of knowledge in general: "it is striking that mathematics itself is not questioned in the empirical studies" (Jung, 1993, p. 144).

Stephen Lerman (1990) writes that

One reads in journals that we should think of mathematics teaching as a process rather than content, but unless the epistemological sources and consequences of this are clarified, and the wider implications for the teaching of mathematics

examined, little will be achieved in terms of development and change in the mathematics classroom (cited in Appelbaum, 1995, p. 9)

Reuben Hersh (1995) urges reframing of the research questions in mathematics education "The issue is not what is the best way to teach, but what is mathematics really about... controversies about High School teaching cannot be resolved without confronting problems about the nature of mathematics" (p. 9). Michael Apple (1995) suggests that "the social construction of what the problem [in math education] is diverts our attention from some of the most important issues" (p. 343). "Let us not act as if the main task is getting a few more students to do well on the cultural capital of elite groups or simply making mathematics more 'practical,'" (p. 346) warns Apple. "Equity, too, may remain a fiction unless we constantly connect it to these struggles for democracy."

In traditional research, "only token attention has been paid by scholars and teachers to the interrelationship between mathematics and society" (Davis, 1993, p. 186). Traditional research in mathematics education has not usually analyzed public policy and its relation to equity in the field (Apple, 1995; Davis, 1992), and it has ignored interrelationships among policy problems, basic design features of a policy and potential equity problems that a policy may help to resolve or exacerbate (Tate, 1995, p. 192). Michael Apple (1995) calls for systemic analysis of 1) inquiry into unequal distribution of opportunity, 2) the role played by mathematics education in social stratification, and 3)

"how to reclaim the aegis of educational reform to include the creation of a fairer social order as a legitimate goal" (p. 333).

Mathematics education research has implicitly constructed a stark distinction between school mathematics and the world outside of schools. Related issues of power and politics, therefore, are placed in the background (Appelbaum, 1995). Focusing on these issues would draw attention to ways in which mathematics and mathematics education practice discursively to construct possible fears and fantasies, contributing to ongoing creation of identities that form an individual. According to Appelbaum, "the conflicts among these identities are crucial" (p. 184). Appelbaum suggests that the aim of mathematics education research be to "probe the arbitrary notions of relevance structured by the very nature of mathematics education as it is commonly perceived" (p. 14).

There has been little communication between those in the mainstream math education research, which has not been concerned directly with equity issues, and those doing research on equity, which has not been concerned with critical mainstream research (Carey, et. al., 1995). Walter Secada notes the tacit assumption that "many individuals lack the capability for engaging in mainstream research" (p. 160) and that is why they turn to equity questions. "Scholarly inquiry that tries to investigate equity group-based differences is seen as derivative (it replicates someone else's students), or is limited to searching for group-based differences" (Secada, 1995, p. 147-8).

The "hidden curriculum" of traditional education has resulted in the following expectations for equity concerns: 1) search for immediate answers, 2) demand for elaborated answers

that fit a preset agenda, 3) exclusive focus on difference, 4) differential standards of scrutiny for equity concerns.

Perhaps the most important goal of mathematics education research should be to "make explicit and articulate the problematique of some of the community's shared beliefs and assumptions about reform and research from the perspective of equity" (Secada, 1995, p. 148). Secada describes a future for research in mathematics education:

Research on equity should anticipate new social questions and new directions in research and policy, rather than lagging behind and then having to play intellectual catch-up, it should question what might come to be, and equity-based inquiry should become an integral part of the agenda from the start. (p. 2)

Finally, traditional mathematics education research has neglected the critically important areas of effective communication and reflection. Michael Apple (1995) suggests that the goal of mathematics education should be "not only the formation of 'critical literacy' in our students but in essence becoming more critically literate ourselves" (p. 343). He adds that educators who wish to make a real difference must continually ask critical questions and must place [effective programs] in their larger context" (p. 346). This concern is directly linked to the ways in which numerous other variables have been ignored in current educational research. Khisty (1995) names some of these key concerns:

"Variables related to teacher discourse, learning environments that promote students talk and educational policies that encourage and support instructional

change in light of these variables have not gotten sufficient attention as we discuss [subordinated] students' performance in mathematics " (p. 295)

Walter Secada (1995) writes that research is needed in classrooms that blends cross-cultural communication and cooperative groupings to understand how such a merger is possible. Indeed, "development of critical dimensions in equity should help to create a discourse and a community that operates at the intersection of multiple domains" (p. 160)

Equity-based analyses should draw on multiple disciplines and voices, appropriate ideas as needed, and remain tentative in what is counted as central to the field.

What is needed is to create ways of talking about research and reform that allow for more textured analyses of phenomena" (p. 161)

Researchers must examine the relationship between educational research and researchers' conceptions of mathematics. Embedded in this issue are a number of interesting and troubling questions. Primary among these concerns is the arrogance that is implicit within the mainstream conception of "mathematics" and its pre-eminence within public schools. Not only has the subject's curricular importance been asserted above all else, nobody has ever seriously questioned *why* school mathematics has been perpetuated as a set of inalienable truths while students continue to die on the battlefield, felled by invisible and abstract weapons launched from all sides. In order to understand the plight of most students, particularly those from subordinated populations, the very construct of "mathematics" must be examined.

Mathematics

"People write about mathematics as if math had fallen from the sky -- Haul mathematics from the Olympian heights of pure mind to the common pastures where human beings toil and sweat" (Struik, 1986, p. 280)

According to Appelbaum (1995), mathematics has been traditionally understood as a body of neutral technology slowly but surely progressing in the service of humanity. It therefore seems hard to ask why and how certain groups become disempowered by mathematics rather than benefiting from its universal gifts. The traditional high school mathematics sequence, culminating with calculus as a final destination, accepts knowledge as predetermined by historical developments in mathematics (ending in the eighteenth century, when calculus was codified). Roland Fischer (1993) claims that "math's meaning became clear" (p. 117) in the late eighteenth or early nineteenth century, when mathematics was viewed as a universal method for explaining and manipulating the world, going beyond the range of the natural sciences. Evidently, "traditional mathematics" has changed very little in the last 300 years despite the consequences of the Industrial Revolution and the more recent explosion of information technologies.

By and large, the mathematics taught in schools may be described as pure and uncontaminated by the real world and taught in a strictly teacher-driven way (Bernstein, 1977). Peter Appelbaum (1995) found that "recent public discourse tends to accept

[mathematics] as a given and static collection of knowledge to be transmitted within schools, transferring pedagogical dilemmas to the site of the individual teacher and his or her transmission of that knowledge" (p. 98). Tharp (1995) found that approximately 50% of all high school students hold a view that learning mathematics is rule-based (i.e. mostly process-oriented and memorization).

Although mathematics could be viewed as a received body of knowledge and skills imparted to a student by an omniscient teacher or other authority figure, mathematics could also be considered to be a communicative discourse developed by groups of people working together to understand problematic situations (Appelbaum, 1995). Ole Skovsmose (1994) describes how mathematics education should be oriented toward three types of knowing: mathematical knowing (including competence in reproducing theorems and proofs, as well as mastering a variety of algorithms), technological knowing (applying math to model building), and reflective knowing (evaluating the consequences of technological enterprises). This final aspect of mathematics education, according to Hans Freudenthal (cited in Skovsmose, 1994), makes it possible for people to underline mathematics as a human activity. However, "no concern has been shown about reflective knowledge in traditional mathematical education" (Skovsmose, 1993, p. 173).

Mathematics, though perceived by some as a set of a priori ideals (an idea linked to Aristotle), can also be recognized as a "human institution" (Davis, 1993). As such, one may recognize the assumption that "what is mathematical is certain" (p. 182). Fischer (1993) describes how, as a human institution, mathematics serves as a means (which humans can handle as a tool) and as a system (which humans must obey and which is

inseparably connected with our social organization) This duality of mathematics (as a means and as a system) is rarely interrogated in research.

Modern mathematics is a social problem in several ways (Restivo, 1993) First, it serves ruling-class interests. Second, it serves as a resource that allows a professional and elite group of mathematicians to pursue material rewards independent of concerns for social, personal, or environmental growth, development or well-being. Third, aesthetic goals can be a sign of alienation or false consciousness regarding the social role of mathematicians. Last, mathematical training and "education" may stress problem-solving rather than ingenuity, creativity and insight. Secada notes that mathematics' "claim to a privileged status are based on basic research's pursuit of universally applicable psychological phenomena without addressing the confounding effects of social context, affect, and the like" (p. 148).

This observation brings into question that nature not only of mathematics, but of knowledge itself. Many see mathematics as the most "pure" of academic subjects; a place where "knowledge" may be most directly transferred from one person to another or to a group of others. The *hubris* that characterizes many mathematicians and mathematics education researchers has strong roots in conceptions of what constitutes "knowledge."

Knowledge

Davis (1993) suggests four common beliefs about the nature of "knowledge" today. Some consider knowledge to be lodged in the mind of God. Others (in a post-

Cartesian age) take knowledge to be an "object objectively known" (p. 188), some loci above or beyond ourselves. Kuhn and Lakatos are known for introducing the concept that knowledge is a socially-justified belief. Davis suggests that, perhaps, knowledge is seen as lodged in the computer! Each of these beliefs is evident in the assumptions underlying mathematics education.

Apple (1995) writes that "part of the problem rests with the epistemological groundings that stand behind not only our definitions of knowledge and knowing (Harraway, 1989), but also our own attempts as researchers to understand these issues in the lives of students, teachers, and ourselves (Gitlin, 1994)" (p. 339). Additional tension arises from the conflict Murrell (1994) describes between mainstream views of knowledge as entity versus subcultural views of knowledge as experiences. In *A System of Logic* (1843), J. S. Mill claimed that numbers in the abstract do not exist. This idea led to the rebirth of empiricism.

Skovsmose (1993) links common understanding of traditional mathematics to the legacy of classical empiricism. In the "rationalistic tradition," knowledge is obtained by an individual activity and knowledge development is separate from communication. Communication is reduced to a pedagogical and a methodological concept. This type of "monological" epistemological theory opposes the view that knowledge construction is dialogical and the result of group interaction. Looking into mathematics classes, illuminated by a monological epistemology, argues Skovsmose, "it becomes obvious that monologism is closely connected to the assumption that a coherent source of knowledge exists" (p. 169). Related to monologism is the idea of "homogeneity of knowledge,"

whereby it is possible to integrate knowledge into one unified system. This epistemology, "logical positivism," was advanced by Carnap, Plato and Russell and it characterizes traditional mathematics education at all levels. In fact, Bertrand Russell viewed the number 2 as a "metaphysical entity" (1956, p. 542).

The epistemology of mathematics characteristic of traditional education is heavily influenced by developments in cognitive psychology. Jung (1995) describes the "nomological-deterministic paradigm" (citing Stangl, 1989, p. 101), wherein the aim of psychology is to discover laws and to explain and predict behavior through those laws. In this paradigm, science and its methodology are recognized to be the norm. The contributions of Jean Piaget (cited in Skovsmose, 1993) support the view that mathematical knowledge is founded in actions (operations) carried out on objects by individuals. Piaget claimed that mathematical knowledge is not created by an abstraction from physical properties of objects; by so doing, he seemed intent to relate both to rationalist and to empiricist tradition. Skovsmose (1993), however, argues that "it is not possible to construct a definition of knowledge without the concept of dialogical relationships" (p. 176). Michael Apple (cited in Appelbaum, 1995) describes the consequences of either tradition. He asks whether assumption of "knowledge" as a psychological "object" or as a psychological "process," thereby depoliticizes the culture that schools distribute. Secada (1995) warns that "we must resist the temptation to transform group membership into psychological terms of discourse and to place social concerns at the margins of the field" (p. 159).

Problem-Solving

Appelbaum (1995) claims that the traditional definition of mathematics as fundamentally "problem-solving" structures curriculum dilemmas and subsequent failures of mathematics education within a "reified, commonsense conception of the discipline" (p 131). In fact, the NCTM Curriculum Standards (1990) have accepted a problem-solving context as resolving any and all epistemological, political, or aesthetic criteria for judging what it is to "do" mathematics. Therefore, the "correct" or "most accurate" epistemology, by prescribing the nature of mathematical knowledge, supplies a stable source of criteria for the development and evaluation of educational programs and associated techniques. Appelbaum (1995) observes that adherence to a problem-solving emphasis in mathematics education buries entrenched disagreements over the nature of the discipline's knowledge content, only to have them resurface within a cyclic pattern of "reform movements" as shifting factions regain political power on a national level.

Contrary to the standard meaning of "problem-solving," Alan Schoenfeld (cited in Appelbaum, 1995) urges that mathematics should be taught not because of its superficial utility but because of its immersion in a problematic context. Fischer (1993) expands on the idea of "problem-solving" that has proven to be so dangerous in mathematics "reform." He proposes that researchers explore the "circularity" inherent in mathematics on one hand, mathematics is used to *construct* a reality and on the other hand mathematics is applied to *describe* and *handle* that reality. This version of problem-solving is very different than that advocated by the Standards documents, wherein students are expected

to solve problems whose answers are already known by the teacher and accepted to be true in all cases

Imre Lakatos established the modern terrain for what is taken to be a fundamental epistemological battle over the philosophy of mathematics, introducing "quasi-empiricism" to counter what he labeled dogmatism, formalism, or Euclideanism. He contended that the unfolding everyday verbal exchange and drama within each real-life classroom is both an enacted argument for a particular epistemology of mathematics and an affirmation that this epistemology drives the enactment itself. Classroom interaction actively constructs an epistemology and human action perpetually reconstructs a commonsense philosophy of mathematics. This debate spurred what appeared to be a major shift in emphasis on communication in mathematics. In fact, this "debate" drew attention away from the social consequences of interaction and centered discussion on the ways in which a pure "mathematical epistemology" could be revealed.

It is interesting to consider the irony implicit in acceptance of the idea that a "debate" about mathematics education has gripped the research community. "Debate" is a word which implies not only formal rules but an effort to strip away assumptions and beliefs that cannot survive without substantiating evidence. One thinks of discussion and questions that lead participants away from initial beliefs into new understandings of issues. However, most "debates" pit currently validated voices against one another. A "winner" is usually identified and participants often must conceal changes in opinion in order to emerge triumphant. Nowhere in all the current talk about the "mathematics education debate" is a concern that other voices, important voices, are being silenced by an ongoing

squabble. It is interesting to consider the ways in which the idea of "debate" so associated with mathematics education policy relates to the similarly bandied idea of "mathematical communication" both inside and outside of the classroom

Mathematical Communication

Critics of mathematics education commonly cite two major problems (Bradley, 1988). First, there seems to be a lack of "meaning" in students' mathematical knowledge. Second, students are unable to communicate using appropriate mathematical language and ideas. "In particular, the ability to think mathematically appears to be the crucial element in mathematics achievement. It may be at this cognitive and metacognitive level that language and mathematics are most intricately related" (p. 5) suggest Dale and Cuevas (1987).

NCTM Curriculum Standards (1990) include "mathematical communication" as a major curriculum strand; however, traditional concepts of "mathematical communication" differ greatly from critical/radical concepts of communication. On one hand, traditional views describe mathematics as a received body of Platonic truths; communication, therefore, is required in order for the teacher to impart these truths to receiving students. Communication serves the purpose of enabling transfer of information from an authority to students. On the other hand, "mathematics" could be viewed as an evolving literacy only artificially distinguishable from other disciplines in that it provides a means for individuals to explain and control complex situations of the natural and of the artificial

environment and to communicate about those situations. Through this lens, language is understood as a socially constructed and ideological in function while practice is a "political and cultural constellation of discourse and behavior modes" (Appelbaum, 1995, p. 51).

Skovsmose (1994) describes two linguistic transitions that must occur if one is to acquire proficiency in mathematical communication. First, one must transition from natural language to a systemic language. Then, one must transition from a systemic language to a mathematical and algorithmic language. These transitions are not recognized in the Standards, where students are expected to develop proficiency in "mathematical language" as a language unto itself. Apple (1992a) differentiates between "functional literacy" and "critical literacy." He concludes that critical literacy, part of a larger social movement for a more democratic culture, economy, and polity, is "decidedly not part of [the Standards Committee members'] agenda" (p. 425).

New conceptions of mathematical knowledge and problem-solving necessitate re-examination of the communication and interaction that takes place in mathematical contexts. Apple (1992a) calls for

a broadened definition of mathematical literacy, one that is used in critical ways to support open and honest questioning of our society's means and ends, is clearly better than a definition that stresses workplace skills and values that largely benefit those with existing power. (pp. 428-9)

Frankenstein (1990) provides a similar concept: "Critical mathematical literacy involves the ability to ask basic statistical questions in order to deepen one's appreciation of particular issues" (p. 336). One of the most significant differences between traditional and critical conceptions of "mathematical literacy" deals with the type of context in which literacy is to be developed. Warren and Roseberry (1995) argue that "mainstream literacies, rather than defining how science is practiced in schools, must be embedded in an entirely different kind of practice" (p. 299) and that education may be better thought of as a process of socialization rather than instruction - into ways of thinking, knowing, and acting that are characteristic to particular disciplines (Resnick, 1988).

Mathematical literacy (called "mathemacy" by Skovsmose, 1994) can become a means for "organizing and reorganizing interpretations of social institutions, traditions, and proposals for political reforms" (p. 39). Sal Restivo (1993) refers to "math worlds," which he describes as "networks of human beings communicating in arenas of conflict and cooperation, domination and subordination" (p. 249). Mathematics is a discipline that is perhaps most distantly removed from the everyday experiences of students. The communication required to be successful in a math class, then, varies greatly from that required for success outside of school. The current discussion about mathematical communication fails to address particular needs of students, especially those from subordinated populations. Schools may be good places to practice mainstream ways of talking and knowing, but they are not good places to acquire them, writes Gee (1990). Khisty (1995) focuses attention on ways in which "learning mathematics involves language

in a way that appears to be much more crucial than previous studies of mathematics education with language minority students might suggest" (p. 283).

A final aspect of mathematical communication requires attention. Walter Secada criticizes advocates of the view that mathematics itself is a language ready for appropriation by others through the process of dialogue-rich classroom activities. Carol Bradley (1988) suggests that "perhaps providing mathematical language activities during instruction can help students bridge the gap between procedural knowledge and conceptual understanding" (p. 22). However, even with a good teacher, writes Secada (1992a), students with limited English proficiency can be "left out" of discourse unless mathematical communication has cross-cultural considerations.

I have never seen anyone ask how using mathematical language for communication or creating discourse communities helps teachers teach diverse student populations. What little evidence I have suggests that the creation of a "discourse community" may result in the exclusion of children of limited English proficiency. (p. 151)

What appears true at the classroom level also applies to interaction among researchers and others involved in formulating mathematics policy. The "mathematics education community" appears to be a much more homogeneous group of individuals than one might assume. Despite diverse personal, geographic and historical backgrounds, members of this community are united by their almost religious adherence to the primacy of mathematics

among domains of knowledge. This common trait has serious consequences for today's schools, their teachers and their students.

The Mathematics Education Community

Davis (1993) asks the question, "To what end do we teach math?" (p. 190) and responds with the following possible answers. Perhaps we teach mathematics because it is beautiful. Perhaps it reveals the divine, or helps us to think logically. Perhaps it is the language of science and helps us to get a job. Perhaps it will allow us to reproduce ourselves by producing future research mathematics and mathematics teachers. Traditional mathematics education exists to prepare students for the next level of mathematics and ultimately for their "proper" vocation (Ernest, 1991; Romberg, 1992). Very seldom would any professor of mathematics answer this question with a response tied to social responsibility or equity. Appelbaum (1995) elaborates on this idea:

Although educators have posited other important areas for consideration other than the discipline itself, they have done so in opposition to the central role of subject area knowledge: either rhetorically, within a strategy for distinguishing mathematics educators from what they perceived as their more highly-regarded colleagues in mathematics, or practically, as they expanded the scope of their professional practice (p. 97)

Some scholars question the significance, apart from sorting, of the predominant operational approach to mathematics (Aronowitz & Giroux, 1991). "Mathematics pedagogy as currently practiced, does not...develop an ability to use and discover mathematical concepts among non-specialists. On the contrary, it encourages the student to learn the logical processes that follow from accepting the axioms of the discipline and its postulates" (Appelbaum, 1995, p. 121).

Many students suffer from adverse experiences in math and science courses and majors in college. This, according to Pritchard (1993) has contributed to the marked decrease in the number of students in math service courses and math majors documented by the NRC study *Everybody Counts* (1989). Although Pritchard cites expected career competition as one reason for this decrease, "another part of the problem lies with the mathematics major itself, which in many institutions has continued a narrow focus on graduate school preparation, ignoring the many students who might (and should) major in mathematics as a versatile preparation for careers in other disciplines" (NRC, 1989, p. 53). Fischer (1993) describes a traditional mathematics education that views the economy as a machine rather than as a living system. Therefore, nobody is asked to think about the whole, which simply functions. The same could be said about the composition of the "mathematics education community" that drives "reform" movements and curriculum "revisions."

Traditional preparation of mathematics teachers reflects many of the same values as traditional mathematics does as a discipline. According to Peter Appelbaum (1995), professors of mathematics education emphasize primarily subject matter expertise and

secondarily pedagogical theory, both over individual teacher attributes Secada (1995) asserts that many barriers to incorporating equity into current reforms can be characterized and traced to particular norms and beliefs that are part of the way that the mathematics education community constitutes itself (p. 3)

Similarly, traditional mathematics education exhibits the consequences of domination of behavioral and competency-based approaches in curriculum development for a long period (based on F.W. Taylor's work in industrial management). Effectively, writes Apple (1995), psychology as the basis for educational research, "eviscerated visions of critical practice" (p. 331)

Traditional "School Mathematics Education"

Despite media coverage of public education issues that lead people to believe that mathematics education is experiencing a radical shakeup, high school mathematics education has remained largely unchanged since the beginning of this century. William Tate (1995) describes what he ironically calls the "foreign method" of traditional mathematics instruction in this country. First, the "foreign method" is characterized by persistent tracking wherein students from subordinated populations have least access to the best qualified teachers. These students also have less access to technology, and they experience significant cultural discontinuity between school mathematics and life outside of school (NSB, 1991, Oakes, 1990, Stanic, 1991). This final characteristic of the "foreign method" is exacerbated in math classrooms, where experience and discussion of life are regarded as "extraneous matters" that are accompanied by penalties. Much of this

cultural discontinuity is due to classroom and teacher assumptions that are closely associated with an "idealized, White middle-class reality" (Apple, 1979) instead of the reality of diverse students.

This underlying set of cultural norms often appears "invisible" in mathematics classrooms, whereas these norms have been examined in other disciplines. "Historically, traditional approaches to mathematics education have been closely associated with a Eurocentric philosophy of elitism and social stratification that aimed to build the economic power and leadership of corporate elites" (Tate, 1995, p. 424). Stanic (1989) asks, "Whose knowledge and whose ways to construct knowledge come to be valued in the culture of school mathematics" (p. 68)? He suggests that the idea of questioning not just the common view of school mathematics but also their own typically unquestioned assumptions about its nature and worth is much more difficult. Secada (1995) explains that the school mathematics curriculum is subject to forces that treat the discipline of math as a symbol and argue for transcendent, cross-cultural universality/neutrality of mathematics.

School Mathematics Pedagogy

Mathematics classrooms are among the most representative of "traditional" instructional approaches. Learning mathematics is thought of as the absorption of discrete pieces of information presented in sequential order. Skovsmose (1995) describes this curricular approach as "structuralism," which views mathematics as determined by

crystallizing fundamental concepts through logical analyses of existing mathematical theories (following Bourbaki)

Traditional high school mathematics pedagogy also has characteristics that can collectively be named the "recitation method" (Tharp & Gallimore, 1991, Secada, 1995). The teacher assigns a text and follows the assignment with a series of teacher questions requiring students to display mastery of material through convergent factual answers. The teacher seeks predictable, correct answers, 20% of which are either "yes" or "no." Rarely are the teacher's questions responsive to student productions. Goodlad (1983) observed similar classroom dynamics. Teachers emphasized note-taking and immediate responses (not unlike game shows!) Teachers talked most of the time and there was little chance for give-and-take between a challenging teacher and learning students. Students assume a passive role and most teachers show little effort to adapt instruction to individual difficulties.

The critical alternative to this traditional "banking" pedagogy might be redefined as "assisted performance" (Tharp & Gallimore, 1991). This approach to teaching calls into question the easy assumption that most teachers are cognitively disposed to facilitate mathematics community learning (Tharp, 1995). In a critical alternative to traditional pedagogy, teachers consider themselves to be "co-investigators" who, with their students, work as "co-researchers" learning to understand their world. However, critical pedagogy is not an easy alternative to traditional banking approaches. Michael Apple (1992a) articulates this challenge well when he explains that the complexity required by critical education will further tax teachers and take away their time and energy. Unfortunately,

most teachers today are currently just trying to "find a way to get through the day" (p. 423).

Clearly, traditional high school mathematics classrooms do not empower students to become creative thinkers or perceptive communicators. Not only are explanations and interpretations, in oral or written form, not a regular feature of instructional activities in mathematics classrooms (Secada, 1995), but "our usual ways of proceeding in mathematics classrooms do indeed hinder the development of our students as responsible persons" (Noddings, 1995, p. 151). Traditional mathematics instruction assumes that some students will learn and that other will not. This perception makes high school mathematics appear ideally suited to the practice of tracking. Students deemed as being less capable are taught less math or are presented with skill-oriented direct instruction and practice as opposed to conceptually-focused instruction promoting problem-solving and understanding (Campbell, 1993).

"School Mathematics" Teachers

NCTM President Shirley Frye said in a 1991 speech, "anyone can achieve confidence in mathematics if properly instructed." However, Appelbaum (1995) explains that this traditional vision of "proper instruction" in mathematics was the explication of easy-to-memorize and easy-to-mimic manipulative algorithms for everyday arithmetical calculations. On the other hand, Henry Giroux criticizes the "de-skilling" of teachers and calls for teachers to be, instead of disseminators of static knowledge, "transformative

intellectuals ” Khisty (1995) criticizes the naive assumption that “anyone can learn mathematics if properly instructed ” She asserts that it is not enough to assume that good teaching is simply good teaching Gloria Ladson-Billings (1995) provides an alternate statement that illustrates a more critical/radical stance “all students can be successful in mathematics when their understanding of it is linked to meaningful cultural referents, and when the instruction assumes that all students are capable of mastering the subject matter” (p 141)

Ladson-Billings (1995) identified characteristics of successful teachers that are rarely, if ever, mentioned in mathematics education reform discussions These include teachers viewing themselves as part of the community and viewing teaching as being passionate and artistic with respect to content rather than aloof and technical Effective teachers “cultivate and maintain strong, interpersonal relationships between teachers and students” (p. 140). This description of effective teachers differs greatly from those attributes valued by professors in mathematics teacher preparation programs.

Consideration of equity as a key component of mathematics teaching philosophy is similarly absent in traditional teacher preparation programs Tharp (1995) suggests that

Since it has been recognized that how teachers view reasoning and mathematics is a key determinant to how they teach mathematics (Simon & Schifter, 1991), it is imperative that educators examine how equity may be engaged in the classroom by teachers who hold varying conceptions of reasoning and learning mathematics (p 3)

Apple (1992a) would agree:

Thinking critically is not necessarily a natural occurrence. Rather, such an awareness is built through concentrated efforts at a relational understanding of how gender, class, and race power actually work in our daily practices and in the institutional structures we now inhabit. (p. 418)

The high school math teacher's role in developing mathematical communication in students also deserves critique. Warren and Roseberry (1995) note that teachers may privilege those students whose ways of talking are the same as theirs. Gloria Ladson-Billings refers to an "abundance of literature on the impact of teacher expectation on students achievement" (p. 137) and Gee (1990) describes ways in which White children are treated like *apprentices* by their teachers while Blacks are treated as *students*. Seymour (1995) reviews extensive literature that revealed differential expectation for boys and girls of mathematics and science teachers (Kahle, 1990) and its effect of generalized lowering of girls' confidence in mathematics abilities (Eccles, 1994; Brophy, 1985; Sadker & Sadker, 1994). These differential expectations have serious consequences. Prior to ninth grade, boys and girls show almost identical mathematical achievement, but after ninth grade, these groups diverge in terms of the number of science and math courses taken as well as academic performance in these subjects (see White, 1992, for an excellent review of related literature).

It is clear that various groups of students experience mathematics education differently. What is unclear is why certain groups of students benefit from the experience

and other groups suffer as a result. Teacher expectation of student achievement appears to be a significant determinant of student success in school mathematics classes. In order to become a math teacher, one must acquire certain skills and beliefs. In mathematics, these skills and beliefs are grounded in a traditional paradigm based on algorithms and the preeminence of empirical reasoning. Math teachers have proven themselves to be successful players in the game of traditional education. They come to high school classrooms steeped in the ideology of traditional mathematics and they prize the same student characteristics and behaviors that advanced their own mathematics careers. It is therefore imperative to examine the ways in which mathematics education functions as a gateway to power and privilege in society.

Mathematics in Social Context

In examining the problem of high school mathematics, Peter Appelbaum (1995) suggests that researchers ask "why the demands of critical pedagogy appear so antithetical to mathematics education and how and why we should or could respond to this appearance" (p. 25). There are a number of reasons why traditional mathematics has remained virtually unchanged in high schools, and many of these relate to researchers' unwillingness to pose questions that relate to the social contexts of mathematics and mathematics education. When taken to be a force actively used by people to maintain power relations in a society, mathematics takes on a whole new meaning than it does when it is taken to be a set of Platonic ideals located above, and beyond, human grasp.

"Until recently, embedding mathematics curriculum and pedagogy within social and political contexts was not a serious consideration; indeed, it was thought of as an almost heretical activity" (Ernest, 1991, cited in Tate, 1995, p. 431). Before the emergence of a few recent critics, the act of counting was viewed as a neutral exercise, unrelated to politics and society (Mellin-Olson, 1987). "Yet when do we ever count just for the sake of counting?" asks Tate (p. 432). Only in math classrooms do we count without a social purpose of some kind (Kietel, 1991). Outside of school, mathematics is a tool used to advance or block a particular agenda (Frankenstein, 1995).

Davis (1993) describes a "mathematized" society and world, where employment of mathematical ideas and constructs are used to organize, regulate, describe and foster

human activities. He explains that the employment of mathematics in a social context is the imposition of a certain order, a certain type of organization not unlike a social contract. The emphasis that tradition has placed on [a Platonic view of math] leads us to shy away from studying the process of mathematization, "to shy away from asking embarrassing questions about this process: how do we install the mathematizations?" (p 184). This inquiry is what Roland Fischer (1993) calls the "didactics of mathematics" to study and shape the relationship between human beings and mathematics by considering mathematics to be a means, a resource, and a system of concepts, algorithms, and rules embodied in ourselves, our thinking, and our actions.

Leder (1995) observes that "educational matters are influenced, if not overshadowed, by the prevailing economic and environmental climate" (p 216). Appelbaum (1995) examined the socio-political environment of the 1980's from which much "mathematics reform" was generated. He concluded that the assumption that epistemology drives educational encounters was injected into public discussions of mathematics education under several separate but mutually-confirming rubrics. One was the dramatic rise of the culturalist wing of the Far Right in the United States. Another was the popular, professional school representations of mathematics that mutually reaffirmed a commonsense understanding of knowledge and problem-solving as static and substantially algorithmic in nature. The conservative discourse in education found a catalytic ally in concurrent concern for linking educational reform to the imperatives of big business (i.e. the SCANS report).

One cannot deny the influence of political and economic events upon the "standard" high school mathematics curriculum when one considers that most members of "standards committees" are political appointees. For example, only one member of the 21-member 1997 California Mathematics Standards Task Force was even a teacher. Apple (1995) documents how the "inner world of education" (the actual processes by which teaching and curriculum planning go on) has been transformed in very damaging ways. These include pursuit of conservative goals, reductive processes of standardization and rationalization (a kind of "factory-izing" of education in the words of Margaret Haley, and systematic de-skilling of teachers)

While mathematics education is considered by conservatives/traditionalists to be separate from the social context in which it evolves, popular knowledge is similarly divorced from the traditional academic curriculum. Nowhere is this more evident than in high school mathematics. Popular knowledge is pathologized, at least in comparison with the existing academic curriculum, which is seen as uplifting and neutral. "The existing curriculum is never a neutral assemblage of knowledge. It is always based on assertion of cultural authority," writes Apple (1995, p. 330). Only once this is accepted can one begin to examine Durkheim's (1961) assertion that individualized thoughts can be understood and explained only by attaching them to the social conditions they depend on

Cultural Discontinuity: School Math vs. Real Math

"We strive mightily - and often fruitlessly - for direct connections between adolescent life and formal mathematics" (Noddings, 1995, pp 152-3) "Mathematics" (as practiced by mathematicians and used by others in their careers and daily lives) is different from "school mathematics" as it is practiced by teachers and students in schools (Stanic, 1989) "Both have a certain 'taken-for-granted' quality" (p. 58) Fischer (1993) notes that among the characteristics of "modern mathematics" (since the later eighteenth century) is that no tight connection between mathematics and "reality" is postulated, mathematics follows a "modeling paradigm." From this perspective, mathematics is not used to *describe* reality, but to *construct* new reality. Traditional modern mathematics, then, in its purest form, is a kind of technology in itself. Similarly, Kline (1962) noted weak links between arithmetic and the "real world," there are special arithmetics for dealing with special situations. Among these are numbers on round clocks (1-12) and Boolean Algebra/logic.

Many studies attest to the alienation students feel from school mathematics (Resnick, 1988, Schoenfeld, 1982). Students perceive "school mathematics as a domain that is disconnected from sense-making and the world of everyday experience" (Secada, 1995, p. 19). The dangers inherent in this kind of dichotomy should be obvious.

Unless the math curriculum includes real contexts that reflect the lived realities of people who are members of equity groups and unless those contexts are rich in the

sorts of maths which can be drawn from them, we are likely to stereotype mathematics as knowledge that belongs to a few privileged groups (Secada, 1991, p 49)

Apple (1992b) suggests that students are being misled if they are told that lucrative jobs of the future will require a high degree of proficiency in traditional mathematics

Appelbaum (1995) asks a similar question "Are we really serving the needs of students by training them in particular algorithms if those algorithms do not necessarily translate into useful job-related skills or meaningful forms of interpreting their world (p 203)? In her book *Mathematical Power* (1993), Ruth Parker asserts that the culture of school mathematics is the antithesis of the culture of mathematics as a discipline: examining the differences between the culture of school mathematics and the culture of mathematics in the real world leads to the inescapable conclusion that school mathematics is unlikely to result in mathematically powerful students

Traditional mathematics education places value on measures of cognitive processes, assuming that these tests provide a valid measure of students' mathematical aptitude or ability. However, "mathematical procedures people use in outside-of-school contexts are often very different from the algorithms taught in schools" (Stanic, 1989), and therefore lower performance of certain groups of these measures reflect cultural differences embodied in the contexts of the assessments rather than real deficits in some basic area of cognition. In light of the differences between school mathematics and real-world applications of mathematical concepts, Stanic (1989) suggests that the "problem"

[with high school mathematics] does not lie in the experience of students but rather in school mathematics itself (p. 63). It therefore behooves people interested in mathematics education to examine the implications of bringing the study of popular culture into the study of mathematics and to incorporate principles of critical pedagogy into the inquiry.

Popular Culture and Critical Pedagogy

"Popular Culture Studies," which attempts directly to address issues of power, identity and politics in relation to schooling, is rare if nonexistent in mathematics education pedagogy. The focal element of a popular culture studies approach to education is critique, which implies the evaluation of one's own practices in relation to the differentiations (and hence inequalities or hierarchies) that they create (Brown & Dowling, 1989). Popular culture represents "not only a contradictory terrain of struggle, but also a significant pedagogical site that raises important questions about the elements that organize the basis of student [and teacher] subjectivity and experience" (Giroux & Simon, 1989).

Locating mathematics in a social context necessitates a "discursive" approach to its study and learning. Following Foucault (1980), Appelbaum (1995) suggests that all mathematical practices be understood as discursive: as readable and criticizable in terms of providing clues to the meanings upon which they are based. Practices become "technologies of the social," sites through which power/knowledge relations are produced (p. 37). Sal Restivo (1995) proposes a "radical sociology of mathematics" in which all talk is taken to be social, a person is taken to be a social structure and intellect is considered to be a social structure.

Rather than emphasizing problem-solving, a critical pedagogy approach to mathematics education emphasizes problem-posing. Grounded in the work of Paulo

Freire (1970), problem-posing is intended to reveal the interconnections and complexities of real-life situations where "often, problems are not solved, only a better understanding of their nature may be possible" (Connolly, 1981, p. 73). Frankenstein (1987) articulates a theoretical framework linking Freire's work to mathematics. This framework must 1) probe nonpositivist meanings of mathematical knowledge, 2) explore the importance of quantitative reasoning in the development of critical consciousness, 3) study ways that math anxiety helps sustain hegemonic ideologies, and 4) probe connections between the specific curriculum and the development of critical consciousness.

Mathematics education that is grounded in critical pedagogy recognizes that mathematics is a site of struggle, a context for interaction among people. Mathematics is never perceived as a pure set of ideals removed from human experience. A critical approach to mathematics education must be aware of "social problems, inequalities, suppression, etc., and it must try to make education an active progressive social force" (Skovsmose, 1994, p. 38). Critical mathematics teachers must develop "conscientization" (critical consciousness) in both themselves and in their students through dialogical, problem-posing interaction with others. Key to the development of conscientization is development of praxis: the interaction of theory, reflection and practice. "Reflection that is not ultimately accompanied by action to transform the world is meaningless, alienating rhetoric" writes Freire (cited in Frankenstein, 1987, p. 182). Fischer (1993) argues that a powerful kind of "social self-reflection" can be supported by mathematics (p. 197).

A critical approach to mathematics education is also based on a dialectical relationship between objectivity and subjectivity. A critical approach seeks to engage the

positivist paradigm, in which knowledge is seen as neutral, value-free, objective, and outside of human consciousness. It challenges the notion that values are the nemeses of facts and must be viewed at best as interesting and at worst as irrational and subjective emotional responses (Giroux, 1981). Ultimately, critical pedagogy allows people to "become aware of the constraints that prevent them from challenging the word, and, finally, to help them collectively struggle to transform that world" (Giroux, 1981, p. 114). Perhaps most important, critical pedagogy is imbued with hope for the liberation of those who have been oppressed.

Our liberation from transcendental, supernatural, and idealist visions and forces begins when the sociological imperative captures religion from the theologians and believers and unmask it. It becomes final (for this stage of human history) when that same imperative takes mind and intellect out of the hands of the philosophers and psychologists. (Restivo, 1993, p. 250)

Power

"Mathematics has been a tool of ruling elites and their political opponents from its development in the ancient civilizations until the present" (Restivo, 1993, p. 274)

Fundamental to a critical vision of mathematics education is recognition of both explicit and implicit ways in which mathematics is used to exert power over individuals

and groups. This concept of "power" differs radically from the notion of "mathematical power" described by the NCTM Standards documents (1990). Not surprisingly, "mathematical power" in the NCTM documents relates to a student's ability to correctly solve mathematical problems and correctly display "mastery" mathematical concepts. "Power" is defined in terms of teachable skills and attitudes. This concept, therefore, continues to construct professional practice that is inattentive to implications of its own role as a "technology of power" (Appelbaum, 1995) and which reproduces dualisms of individual/society and school knowledge/popular culture. Why has this notion of power not entered recent public discourse on education? Appelbaum (1995) suggests that this is because most educators who have made substantial use of the concept are liberal or left-leaning academicians who are closed off by a conservative public philosophy. This supports Foucault's assertion that sites of "power/knowledge" are historically constituted rather than being the product of universal or transhistorical structures.

Other elements of Foucault's conception of "power/knowledge" apply to mathematics education and its role in society. "Mathematical power" can be construed as the energy behind social, economic, or cultural movements. It may be perceived as a scarce commodity, or, as asserted by Foucault, it is the property of relationships, similar to the role played by gravity in physics. Power, therefore, is a property of all relationships and practices and structures the field of possibility for action (Foucault, 1979). The "standard" definition of "power" relates in no way to students' engagement with the discipline of mathematics, teachers, or other aspects of the educational system in which they must operate. Appelbaum (1995) compares NCTM's stated intent to

"mathematically empower" students to the way one services an engine mathematics must feed workers so they may run the economy, provide lifelong maintenance so that the engine is a long-term investment, and make use of all components so that none may be wasted.

Valerie Walkerdine (1981) applies Foucault's paradigm by naming education and other practices as "technologies of the social." This suggests that the practices themselves become sites through which power/knowledge relations are produced and are part of the apparatus of government and regulation of the population. Mehrtens (1993) asserts that the social legitimacy for mathematics is basically attained in two ways: through the utility of mathematics and through its cultural value. According to Mehrtens, mathematics defines its identity through the specificities of its knowledge. "Purity" thus serves an extremely important function by producing and reproducing identities of a system and important elements of the identities of the individuals taking part in the system.

Students in school mathematics classrooms seldom exhibit behavior of empowered persons. The contrary, a "mathematically disempowered" person, sees problem-solving as an exercise in finding some key-word clue that indicates what operations or formula should be used to transform the given numbers into the expected answer (Frankenstein, 1995). Perhaps the first observation that can be made about the climate in school mathematics classrooms is that students feel powerless. "Fully human beings have some control over the conditions in which they work, the evaluation of their own efforts and the level of reward for which they are willing to work," observes Noddings (1993, p. 151). In a mathematics classroom where students regularly face multiple-choice tests and an

abstract curriculum over which they have no control, they surely must feel less than "fully human." Perhaps this is why students express such a dislike of mathematics.

Critics of education often cite a dichotomy between developing students' "content-area mastery" and developing students' "self-esteem." Critical educators see no dichotomy. In fact, self-esteem must be considered a measure of confidence and power over forces that would otherwise leave a person disempowered. Like Freire (1970), Giroux (1989) observed that this is the goal of critical literacy. Skovsmose (1994) asserts that "critical education can refer to the claim of an equal distribution of possibilities for education within a democratic society...and it can imply a concern for the self-confidence of the students" (p. 51) since the two are inseparable.

Students often blame themselves or some outside force for failing in mathematics. Although they say they want to do well in mathematics, they can't. Teachers cite a number of reasons for this problem; mathematics is the subject most students fail. Few respond to the problem as Marilyn Frankenstein (1987) does when she observes that "people with the most dominated, 'semi-intransitive' consciousness have a fragmented, localized awareness of their situation and are unable to think dialectically about it" (p. 184). Seldom do mathematics teachers address students' need to engage the subject dialectically and therefore gain power they need to "succeed." One way teachers either intentionally or unwittingly disempower their students is by denying them access to data and information that might allow them to explode "myths" (Frankenstein, 1995) about social factors including class structure.

"Power" from a critical standpoint is generated and maintained through interaction with others. This idea is not unlike Hannah Arendt's conception of power as a human ability to act in concert with others, it is never the property of an individual. David Nyberg (1981) notes that "consent holds power together." It is therefore the responsibility of critical educators to maximize the possibilities of power as a conceptual tool in the formulation of a philosophy of education oriented toward practice.

Ideology

From Pythagoras in antiquity to Bourbaki in our own days, there has been maintained a tradition of instruction - religion which sacrifices full understanding to the recitation of formal and ritual catechisms, which create docility and which simulate sense. All this has gone on while the High Priests of the subject laugh in their corners. (Baruk, 1985)

Antonio Gramsci (1971) described ideology as immersed in something so completely total and lived at such a depth that it saturates society to such an extent that it constitutes the limit of common sense for most people. Closely linked to the power/knowledge role played by mathematics and mathematics education is its utility as an instrument of ideology. Nowhere is this more evident than recent developments in the "Standards Movement" and in "official" documents intended to drive mathematics "reform."

Louis Althusser (1971) describes ways in which individuals are "interpellated" by ideology: through ideology, a person identifies him/herself as who they are. Schools are "industrial state apparatus" institutions by which the state wields power over the interpellating role of ideology. Ole Skovsmose (1994) describes the "formatting power" of mathematics and suggests that researchers examine the impact of the mathematics research paradigm on technology and society. He notes that mathematics freezes an algorithm for what to do, and in that sense the (implicit) mathematical model becomes a guideline for action. Inherent in this process is a kind of objectification that is a general aspect of mathematical formatting.

Fischer (1993) adds to this idea by arguing that the function of mathematics in the process of collective self-reflection is objectivization (sic) in the sense of reification wherein social and other relations are made objects (p. 212). This "freezing" aspect of the traditional mathematics research paradigm is obviously better suited to conservative interests than it is to critical/radical interests. Critical educators, then, must strive to make the process of objectification (and the construction of symbols) more explicit and make it available to people (especially students).

The linguistic aspects of mathematics must be critiqued from an ideological perspective. Bakhtin (1981) and Gee (1990) observe that discourses are inherently ideological and that they crucially involve a set of values and viewpoints in terms of which one speaks, acts and thinks. Therefore, according to Warren & Roseberry (1995), "appropriating one discourse will be more or less difficult depending on the various other discourses in which students and teachers participate" (p. 309). Along these lines,

Frankenstein (1995) comments that "the use of mathematics in everyday life not only makes our choices seem more 'rational,' but also serves to end all discussion" (p. 168). Skovsmose (1995) asserts that "the ideological function of monologism is to rule out the importance of dialogue, and, by doing this, the possibility for knowledge conflicts being the starting points of reflections will become eliminated" (p. 177). As a result, mathematics results in "cultural imperialism" conveying the essence of the rationalistic tradition integrated into the social and political structures of highly technological societies.

Hegemony and Counter-hegemony

Raymond Williams (1976) explicates the work of Antonio Gramsci:

[Hegemony] is a whole body of practices and expectations; our assignments of energy, our ordinary understanding of man [sic] and his world. It is a set of meanings and values which as they are experienced as practices appear as reciprocally confirming. It thus constitutes a sense of reality for most people in the society, a sense of absolute because experienced [as a] reality beyond which it is very difficult for most members of a society to move in most areas of their lives (p. 205).

Popkewitz (1987, 1988) describes how mathematics serves to give symbolic reference to the science/technology base of society, providing higher status to experts who have mastered the subject. Mathematics also obscures the socially-constructed nature of many phenomena studied via mathematics. These factors create and support social forces that

help shape school mathematics and that enable reform to operate on both symbolic and technical levels.

School mathematics plays a number of hegemonic roles in society. Perhaps the most significant contributor to the development of mathematical power by certain groups is the widely accepted notion that schools are organized according to merit. Students are expected to blame themselves for failure and to accept their eventual placement in low-status jobs as the natural outcome of their own shortcomings (MacLeod, 1987, p. 113). Apple (1992a) observes that mathematical and scientific knowledge are forms of "high status knowledge" (p. 420) in society because business and industry (as well as government) place a high value on knowledge that is convertible ultimately into profits and control. Traditional mathematics education and mathematics education research in general fails to acknowledge the ways in which our educational systems are organized to ultimately produce, not distribute, particular kinds of knowledge that are needed by business and industry and for the defense establishment (Apple, 1985).

In general, our educational system ritualizes all students as equal. Those with natural ability pass successively discriminating tests and emerge as a highly-qualified few fit (by nature) for high-income jobs and positions with high social power and influence (Fiske, 1987). The dominant ideology insists that everyone has a chance to rise up through the class, economic, and power systems. Those who do not rise (by definition, the majority), have failed through their own natural deficiencies. Thus the high school curriculum embodies a "selective tradition" (Apple, 1979, 1982, 1988, Bernstein, 1977; Bourdieu and Passeron, 1977, Giroux, 1983, Skovsmose, 1995, Stanic, 1989, Taxel,

1989, Whitty and Young, 1976), that provides the greatest benefits to powerful groups in our society and serves to reproduce the social inequalities that we explicitly ask our schools to overcome.

Due to the particular and powerful role of mathematics in perpetuating this "selective tradition," adherence by public discourse to mathematics as a central, unique component of an education allows the continued acceptance and evolution of mathematics education as an important subspecialty of pedagogy (Appelbaum, 1995). The mathematics curriculum, like curricula in general, is conditioned by social functions of education in a stratified society. Educational institutions in advanced industrial societies "foster types of personal development compatible with the relations of dominance and subordinancy in the economic sphere" (Restivo, 1993, p. 264).

This "selective tradition" nurtures development of groups of specialists and experts who are "outside ordinary democratic control and outside the competent 'discourse' in society" (Kietel, 1991). Romberg (1992) warns of the danger that the democracy could be replaced by an expertocracy with technical knowledge of mathematics as well as the reflective knowledge necessary to analyze and evaluate technological developments. The link between high school mathematics and development of this "expertocracy" is clear. As Bowles and Gintis (1976) explain, the rule-orientation of the high school reflects the close supervision of low-level workers, the internalization of norms and freedom from continual supervision in elite colleges reflect the social relations of upper-level, white-collar work.

Several reports published by the Educational Testing Service (1990, 1991) document a "strong relationship between students' economic status and the level of

resources provided for their classroom experiences" (Tate, 1995, p. 437). Among the most hegemonic functions of school mathematics is the widespread practice of tracking students by placing them in various math courses in junior high school or middle school. Tracking is an explicit example of the social stratification that originates in mathematical contexts.

Whereas many mathematics "reform" documents call for mathematics that is used in "real life," persistent tracking evokes the question of "whose life" is to be considered. Apple (1995) criticizes the sloganizing of the term "mathematical literacy," supposedly required for flexible job performance. Contrary to the useful rhetoric that students will need more math if they hope to acquire well-paying jobs, Apple (1995) notes a steady movement toward low-wage, part-time, nonunionized, no-benefits, service-sector employment. The tracking that begins in middle school is indicative of school mathematics' function as a "critical filter" (Leroux & Ho, 1993), which determines entry into a host of careers requiring mathematical skills. Educational tasks and social relations are much more mechanical and formulaic in working-class schools, so these students are tracked out of higher schooling and jobs (Anyon, 1980; Weis, 1990).

Not surprisingly, many students respond to the hegemonic function of traditional school mathematics education with resistance. It is interesting to consider Foucault's observation that resistance is a necessary fuel for the maintenance and concretization of existing power relationships; perhaps traditional mathematics education requires student resistance for its very survival. There is substantial literature analyzing the alienation from school felt by many students because they have lost faith in the capacity of school to

prepare them for adult life (Metz, 1988, Fordham & Ogbu, 1986, Secada, 1995)

Frankenstein (1990) observes that many students do not realize that they already know and use a lot of basic mathematics in their daily lives as consumers and workers; instead, they have internalized the "reified typification of mathematics" (Spradberry, 1976, p. 240) and have the same conception of mathematics that their previous teachers and texts presented (even though they have been unsuccessful in learning or remembering "school mathematics").

Why are these concerns conspicuously absent from the current "debate" over school mathematics reform? How can reformers focus exclusively on the "content" students are to "master" if there appears to be such significant grounds for student resistance? Clearly, the "standards" movement which is securing its grip on mathematics education at the state and national level is not informed by the concerns outlined in this section of the paper. Whose interests have informed this movement? Who are they players? What are the characteristics of the traditional philosophies underlying this movement? These are questions addressed by the following section.

Standards and Mathematics Reform

In response to public outcry about the poor state of mathematics education in the United States, the National Council of Teachers of Mathematics began what would be a six-year process of Standards development. These documents included *Curriculum Standards* (1989), *Professional Standards for the Teaching of Mathematics* (1991) and *Assessment Standards* (1993). Development of these documents was driven by a desire to establish "accountability" that could be used to improve the condition of mathematics education in the United States. Release of the SCANS report (1994), which outlined those skills and characteristics businesses expected to find in their employees after graduation from high school, coincided with release of the NCTM documents. This suggests that the "accountability" advocated by NCTM was significantly aligned with the "accountability" outlined by business. It is therefore essential that people interested in mathematics education research ask what such documents might look like if, as Shaw & Miles (1979) suggest, accountancy in terms of money and profit were replaced by accountancy in terms of social need.

Clearly, the composition of policy-making bodies affects the policy generated. Although the NCTM Standards development boasted diverse committees of writers, Apple (1992a) suggests that within diverse groups those with the most intrinsic power will exert the most leadership, even tacitly. "The most powerful elements, the groups exerting the most leadership and having the most impact, were part of a largely rightist coalition"

(p 438), writes Apple, who adds that "It is not the largely meritorious motives that stood behind the Standards that will determine the uses to which they will be put. It will be the current social context, a context increasingly defined by the right" (p 438). Secada (1995) summarizes the consequences of the silent voices

Given the absence of any significant role for minority educators in the formation of the NCTM Curriculum and Evaluation Standards (1989), or the Professional Standards for Teaching Mathematics (1991), the power issue may loom large in attempts to implement NCTM goals in urban, predominantly minority communities (p 51)

The NCTM *Curriculum Standards* (1989) state that "Creating a just society in which women and various ethnic groups enjoy equal opportunities and equitable treatment is no longer an issue (p. 4). George Stanic (1989) questions this definition of "just society." Is this one based on an economic necessity rationale (as suggested by the Standards), or one based on moral necessity? He concludes that "equity has never been the central theme of a major reform effort in the history of mathematics education, and, not surprisingly, it has never been achieved" (p. 60). This is not to say that the framers of the Standards documents intentionally sought to ignore equity, as Stanic explains, "the authors of the *Curriculum and Evaluation Standards* decided that the best way to achieve equity is to have it as a goal but not to focus on it" (p. 60)

An interesting, and troubling, characteristic of the NCTM standards and GOALS 2000 (the federal education policy born in the late 1980's) is the effectively naive rhetoric

that schools should enforce "high standards for all students." As William Tate (1995) explains, this emphasis on high standards for all students effectively diverted attention away from underserved students, and as Stanic (1989) reminds, "focusing on what is apparently good for everybody has, historically, never been good for everybody" (p. 60). Tate (1995) cites three reasons explicitly given by NCTM for development of the Curriculum and Evaluation Standards (1989): First, they would help to ensure quality education. Second, they would indicate goals for student learning. Last, they would promote change. However, policy proposals linking fiscal adequacy and opportunity to learn considerations to the new math standards faced political opposition. Was "quality" taken to mean the same thing as the 3.99 average GPA cited as "quality" by the University of California when under fire in 1998 for its drastic reduction in the number of "minority" students admitted? Just how much "change" was envisioned in the first place?

Secada (1989) critiques the reform documents in terms of "enlightened self-interest" versus genuine pursuit of social justice. He focuses on the catchy, meaningless phrase "math for all" that runs through these documents. The emptiness of this phrase is illustrated by the prospect of realizing the Standards' call for "technology rich classrooms." Schools across the United States are already characterized by vast fiscal disparities; the funds required to enrich classrooms with technology will, not surprisingly, be allocated according to previously-determined policies that will exacerbate existing gaps. As a result, students "who are situated to take advantage of educational innovations will receive a disproportionate amount of their benefits" (Secada, 1989, p. 40). Even "voluntary mathematics standards will make it nearly impossible - simply for economic

reasons - for poor urban districts to have enough money to support them" (Apple, 1995, p. 339)

The current Standards discussions (that revolve around State and National assessments that will ultimately be tied to funding by both sources) are heavily influenced by the Clinton administration's emphasis on individual "responsibility" (Tate, 1995). Few proponents of the Standards (and accompanying "reforms") seem aware that "unless equal investments are made to improve classroom and school conditions that ensure opportunity-to-learn, these reforms are likely to have little lasting impact and will become punitive to the students and teachers within them" (Winfield, 1993, p. 307). This is especially true for communities of people from subordinated populations. Opportunity-to-learn must be viewed as a major equity concern for students from low-socio-economic or racial/ethnic minority groups (Stephens, 1993).

Perhaps the most serious problem with the Standards documents that currently drive mathematics "reform" is their conspicuous failure to focus on equity. If equity has never been an essential consideration in mathematics education research, how can one expect it to appear as a central consideration in mathematics reform documents? Mathematics "reform" has been reduced to issues of curriculum and instruction, as if mathematics educators could really affect a school's curriculum and teaching without taking into account the entire school! Secada (1995) explains that

People want answers that are immediately and easily applicable, are elaborated and fit into current research paradigms and pre-existing belief systems, focus

exclusively on differences, and can meet standards of evidence and scrutiny beyond those that are applied to their other beliefs (p. 149)

As a result of these expectations, the rush to reform means that when an issue of equity gets raised it must be solved almost immediately. Unfortunately, the results are often superficial - "symbolic changes that fail to address the real issues at hand. Sometimes it is easier [for equity advocates] to be silent" (Secada, 1995, p. 149)

Politics and Policy

If "educational policy brings together resources of government - money, rules, and authority - to achieve political objectives" (McDonnell & Elmore, 1987), then mathematics education policy must be inherently political as well. Peter Appelbaum (1995) suggests that "we need an antidote to the commonsense construction of [mathematics] as a neutral apolitical realm" (p. 196) and advocates a kind of Foucault-inspired "intellectual terrorism" in the mathematics education community. Skovsmose (1993) makes a similar recommendation: "to overcome a [monological perspective of mathematics], it is necessary, from an epistemological point of view, to identify the political dimension of mathematics education."

Despite supposed "mathematics reform," the political dimensions of mathematics education have remained largely untouched. Indeed, "the sacred position of mathematical

knowledge is the basic epistemological cause of the invisibility of its political dimensions” (Skovsmose, 1993, p. 172). This sacred position has been preserved in spite of - perhaps because of - relatively superficial attempts to “reform” mathematics education. One example is the “constructivist” movement, which many critics view as a revolutionary change in pedagogy. Unless constructivism is embedded in an encompassing moral, ethical, and political educational framework, it risks categorization as a method, as something that will produce enhanced *traditional* results (Noddings, 1993, pp. 158-9).

A similar argument could be made regarding the widely-voiced call that students take more math courses (“rigorous” University of California application requirements are one example). “It will be insufficient for students to take more math courses if those courses teach content that is too limited, if they fail to connect mathematics to students’ life experiences, and if they fail to empower students to use math in a wide variety of settings” (Secada, 1995, p. 22). What mathematics “reform” should be about is critically examining the nature of the subject, not requiring students to “take” more of the same subject that is already so problematic in many ways. It is important that mathematics educators and those interested in mathematics education research consider what Tate (1995) calls the “normative” component of educational policy: policymakers’ convictions regarding how the system *ought* to work are based on values associated with educational policymakers’ ideology or political philosophy. “Policymakers prefer policy instruments consistent with their own values” (p. 196).

Finally, in asking new questions about the political aspects of mathematics education, one must critique the making and use of slogans (see Apple, 1992a) and labels

used to sort and rank students (see Frankenstein, 1987) It should be noted, as does Maxine Greene (cited in Appelbaum, 1995) that NCTM's definition of "mathematical power for all students" is characterized by the ambiguity that is essential to all slogans, and that "uniformly high expectations for all students" is part of a sloganized attempt to make schools look democratic and egalitarian (Noddings, 1993, p 159). This, perhaps, is one of the best places to start when critiquing recent movements called mathematics "reform."

Chapter Three: Methodology

This study suggests that "high school mathematics," if viewed through a critical lens, looks very different from that accepted by the traditional perspective evident in even the most "progressive" *Standards* documents. This study seeks to illustrate the "high school mathematics" that is revealed by peering through a critical/radical lens. Part of this process involves problematizing assumptions otherwise taken for granted by mathematics education researchers.

Investigative Approaches:

To this end, four distinct yet connected methodologies are used to problematize "high school mathematics."

1. First, 1205 of the 1440 students in a low-SES high school populated by a majority of students of color respond to a **written survey** that seeks to address 1) students' value of mathematics, 2) students' mathematical confidence, and 3) students' resistance to mathematics. Demographic information and self-reported math grades are included in the analysis of survey results. This survey data provides the means of asking questions about the epistemological and social aspects of high school mathematics that currently do not appear in mathematics education research.

2. **Second, each mathematics teacher at this high school provides the names of three kinds of math students** "good" math students, "average" math students, and "poor" math students. Teachers are asked to provide the names of two students in each category and also to explain their understanding of what characterizes "good," "average" and "poor" math students. Teachers are interviewed to obtain this data, which is used for the purpose of assembling interview groups and collecting student narratives.
3. **Third, five mixed groups of three students** (identified by their teachers as "good" math students, "average" math students, and "poor" math students) **participate in interviews as focus group members**. Structured interview questions address hegemonic functions of mathematics, student resistance to mathematics and the objectifying function of "high school mathematics." Narratives resulting from both student responses to specific questions and discussion among interview group members provide data that illustrate qualitative aspects of data obtained from statistical analysis of student surveys and teacher comments.
4. **Fourth, academic transcripts of participating students are analyzed**. Among the factors considered are math GPA, English GPA, citizenship GPA, grade level, writing sample score, standardized test scores, and credits earned. Transcript data are used to illustrate the role of high school mathematics in the public education system experienced by students in one high school representative of those populated by students of color from lower- or lower-middle-class socioeconomic backgrounds.

Research Questions

The study responds to the following questions:

- 1 What factors contribute to a student's success in high school mathematics?
- 2 How do various groups of students experience high school mathematics differently?
- 3 How are slogans and ideology employed by high school mathematics education?
- 4 How does tracking reflect the hegemonic role of high school mathematics education?
- 5 How is high school mathematics characterized by student resistance?
- 6 How does interaction/dialogue in the mathematics classroom affect student confidence in high school mathematics?
- 7 How does high school mathematics function to "objectify" students and teachers?

Table 1 matches each investigation to its guiding research questions, procedures, population, instruments, constructs and analysis

Table 1

	Investigation #1	Investigation #2	Investigation #3	Investigation #4
Research Question(s)	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 7	1, 2, 3, 4, 5, 6, 7	1, 2, 4, 5, 7
Procedure	Statistical Analysis	Structured Interview	Focus Group Interview	Transcript Analysis
Population	n = 1206	n = 12	n = 18	n = 52
Instruments	Student Survey	Interview notes	Audiotapes, transcriptions	Transcripts, Interview notes
Constructs	resistance, confidence, value of math	mathematical ability, teacher expectations	student perception of mathematics and math education	systemic role of math education, ideology
Analysis	SPSS Descriptive. Path Analysis. Analysis of Variance. Regression. Factor Analysis	Content Analysis. Ethnographic Techniques	Stanzaic-Poetic Narrative Analysis. Thematic Narrative Analysis	SPSS Correlation. Descriptive. Qualitative description

Investigation #1

Investigation #1 is designed to provide responses to research questions 1, 2, 3, 4, 5, and 6. It relies upon statistical analysis of student responses to a survey dealing with both "objective" and "subjective" information related to students' experiences with mathematics education (see attached surveys). For 30 statements related to mathematics education, the survey asked students to mark their opinions on a Likert scale ranging from 5 (Agree) to 1 (Disagree). Additionally, the survey asked students to indicate their current mathematics course, their grade level, their gender, and the scholarship they "usually get" in their mathematics classes.

The survey was developed during a three-week period during the Spring semester of 1998. Several mathematics teachers at Mar Vista High School contributed feedback regarding what they thought were interesting questions and reviewed the wording of the statements. Several minor changes were made to the original survey. One of the mathematics teachers translated the final version of the survey into Spanish and again reviewed its content. A second bilingual teacher edited the survey for accents and spelling and provided further comments. Finally, the two versions of the survey were duplicated according to the number of students currently enrolled at Mar Vista High School (regardless of whether or not these students appeared to be registered for a mathematics course at the time).

The survey was tested for internal consistency with a group of 38 students currently enrolled in a Course Two class. Analysis of intercorrelations among specific

Likert scale items supports the assertion that the survey measured at least three constructs. The first of these constructs, "student resistance to mathematics" is defined by student responses to statements 29 (I feel afraid when I'm in my math class), 16 (I feel angry when I'm in my math class), 24 (I feel helpless in my math class), 20 (I can't do math), 3 (I hate math), and 12 (math has nothing to do with my life). The Cronbach's alpha among these items was .77. The second construct, "student confidence in mathematics" is defined by student responses to items 18 (I expect to use math when I am an adult), 17 (My math teacher thinks I am a good student), 4 (It is important to do well in math classes), 6 (Math is the most important subject in high school), 28 (I feel happy when I'm in my math class), 10 (I look forward to working on math problems), 11 (Math is my favorite subject) and 2 (I am good at math). The alpha among these items was .84. The third construct, "students value of mathematics" is defined by student responses to items 4 (It is important to do well in math classes), 6 (Math is the most important subject in high school), 13 (People who do well in math are smart), 7 (People who do well in math will succeed in life), 18 (I expect to use math in my life when I am an adult), and 1 (I understand why I have to take math in school). The alpha among these items was .82 (see Table 2).

Table 2 Major survey constructs

	"Resistance"	"Confidence"	"Value of Math"
Cronbach's alpha	.77	.84	.82

The Pearson correlation between "resistance to mathematics" and "student confidence in mathematics" was -.56. The correlation between "resistance to

mathematics" and "student value of mathematics" was $- .35$. The correlation between "student confidence in mathematics" and "student value of mathematics" was $.73$. None of these correlations is so high that the constructs may be considered to be the same.

The school registrar provided a class-by-class list of how many students were enrolled with which mathematics teachers. He also provided a list of students who were not currently enrolled in a mathematics course. Each mathematics teacher received a cover letter (see attached) explaining the purpose of the survey and outlining the request that surveys be distributed and collected in a timely manner. Teachers were asked to report the total number of students in all of their mathematics classes as well as the number of surveys that were returned. Students not currently enrolled in mathematics were individually called out of class and asked to respond to the survey. 1377 students were currently enrolled in mathematics classes and 47 were not. Of the total 1424 surveys that were distributed, 1262 were returned and 1206 were determined to be valid. This reflects an 89% return rate. Since some of the returned surveys appeared suspect (i.e., a response of "1" appeared beside each of the Likert scale items), 85% of all the distributed surveys were considered valid and therefore used in the investigation.

To gain some measure of the validity of students' self-reported scholarship grades in mathematics, students' self-reported grades were correlated with official mathematics course grades as of October 1997 (the semester preceding the semester during which this study took place). Since it was impossible to match individual self-reported grades with individual official mathematics grades, and since students were asked to report the "scholarship grade" they "usually get" in their math classes, average grades were

calculated for students in each mathematics course offered. The Pearson correlation between self-reported grades by course and official school mathematics grades in October by course was .97.

The correlation between the number of students in each mathematics course as indicated by students on the survey and the number of students officially enrolled in each mathematics course as indicated by school records was .99. These correlations support the assertion that self-reported survey data describing students' grades and course placement matches data that might be obtained for individuals from "official" school sources (i.e. databases, etc.) and that the survey results are representative of all students in the school (see Table 3).

Table 3: Self-reported and Official grades and numbers of students

Math Course	Self-reported math GPA 3/98	Official math grades 10/97	Number of surveys per course 3/98	Students enrolled in each course 10/97
Math A	2.29	1.10	87	117
Math B	2.11	1.09	37	53
Course 1	2.58	1.53	332	416
Course 2	2.48	1.70	267	391
Course 3	2.61	1.99	185	225
Pre-Calculus	2.96	2.41	45	61
Calculus	3.30	2.90	23	20

Responses to student surveys were entered into an SPSS database over the course of two weeks. Three people performed the data entry. Once all raw data was entered, it was checked for spurious or out-of-range values. Each variable (and its possible values)

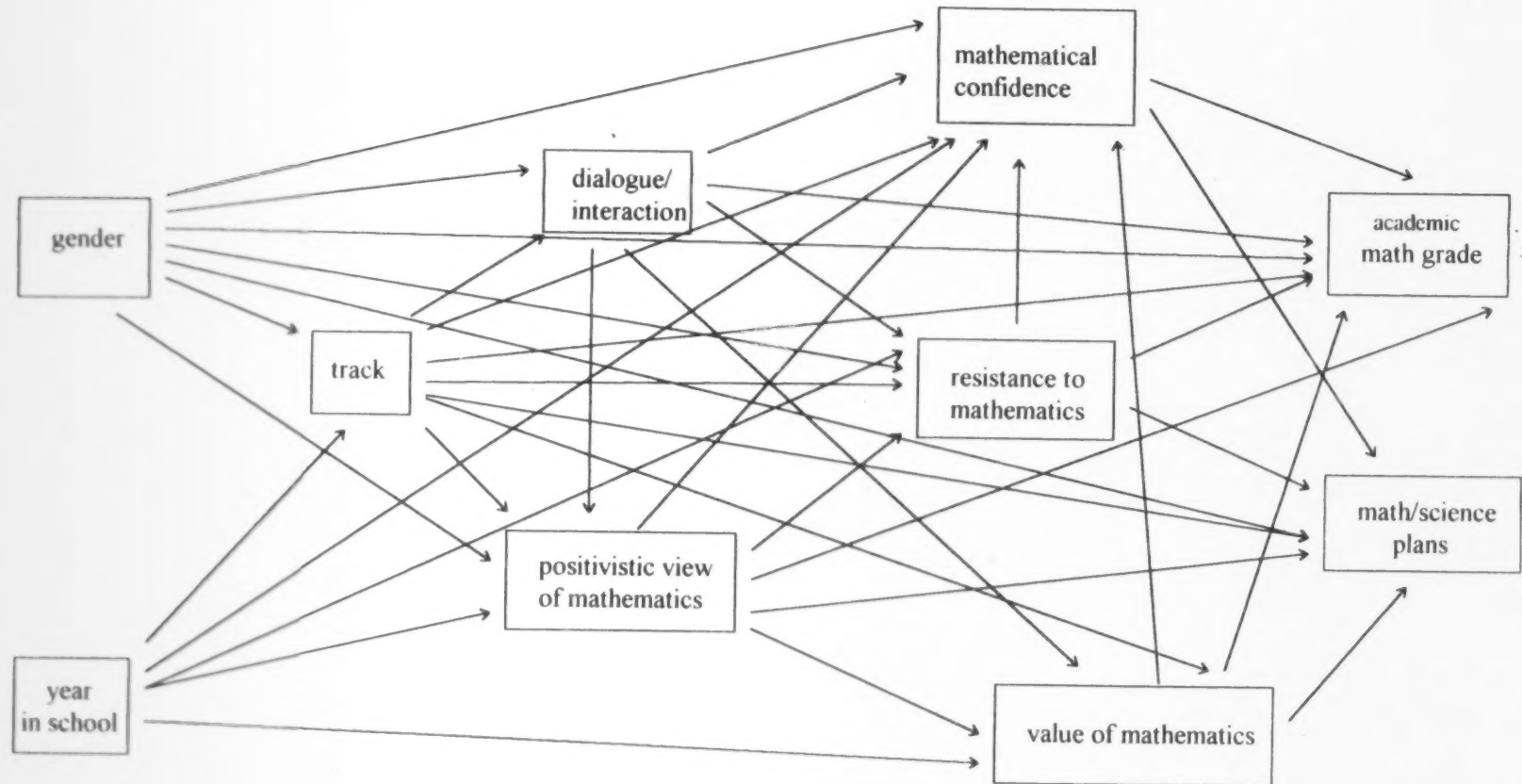
was coded and labeled. Composite variables ("Resistance," "Confidence," and "Math Value") were added by calculating the mean of students' responses to the contributing items. The variable "College Prep" was added by dichotomizing students' math classes into either college prep (Course 1, Course 2, Course 3, PreCalculus, or Calculus) or non-college prep (Math A or Math B). The variable "Object," consisting of the average of students' responses to item 22 (The way to succeed in math is to memorize things) and item 23 (Answers to math problems are either right or wrong) is based on the overall Pearson correlation of .78 between these two items. This variable describes the "objective" nature of mathematics as perceived by students.

The variable "Interact" is composed of the average of students' responses to item 14 (I often hear conversations about math in my math class), item 21 (I often work with other students in groups to do math), item 15 (My teacher asks me to explain my answers to math questions), and item 9 (I am expected to stay quiet in my math class, which was recoded) since all questions related to the degree of interaction a student experiences in school math environments. Students who are enrolled in classes which receive special services due to a CAPP grant are coded as "CAPP" students.

Several statistical analyses were performed using this database. A path analysis explores the causal relationships among the composite variables "Resistance," "Confidence," "Math Value," "Interaction" and "math grade" as the ultimate dependent variable (see Figure 2). Analysis of variance explores ways in which student responses to various items differed in terms of math course and grade level. Analysis of variance also explores differences between the responses of students enrolled in "college-prep" math

courses to those enrolled in "non-college-prep" math courses. Male/female differences are examined, as are a variety of correlations. Finally, descriptive statistics suggest general student perceptions about the nature of mathematics and math education.

Figure 2: Pre-Analysis Diagram



Investigation #2

Investigation #2 is designed to provide responses to research questions 1, 2, 3, 4, 5 and 7. It relies upon feedback from high school mathematics teachers to develop a profile of what teachers consider to be "good mathematics students," "average mathematics students" and "poor mathematics students." Mar Vista High School math teachers were asked to give input during one week in February 1998. Each of the twelve members of the mathematics department was asked to nominate six students who might participate in focus group interviews regarding the nature of mathematics education.

Teachers were asked not to nominate only talkative students but to make their nominations based on their definition of "good," "average," and "poor" students. They were also advised that they might nominate students with a primary language other than English. Teachers were asked to provide the names of students in any of their mathematics courses; some assumed that the "good" mathematics students had to be members of their more advanced math courses. Ten teachers provided information via oral interviews; two chose to provide their input in writing. All teachers (twelve of a possible twelve) chose to participate; data collected via this investigation may therefore be seen as representative of the school's general "math teacher expectations."

A notetaking guide (see attached) was used to record the names of students, their courses, and the characteristics of "good," "average," and "poor" mathematics students. Most teacher interviews lasted approximately twenty minutes; the shortest one was ten minutes in length and the longest one lasted fifty minutes. Teachers provided thoughtful

commentary regarding the student characteristics which contributed to success in their math classes.

Investigation #3

Investigation #3 is designed to provide responses to research questions # 1, 2, 3, 4, 5, 6 and 7. It relies upon focus group interviews with high school students in grades nine through twelve. Four groups of students met for hour-long interviews during one week in March, 1998. The smallest of these groups consisted of four students, the largest consisted of six students. Interviews were tape-recorded and notes were taken during the interviews themselves.

Students were selected based on teacher nominations. During the previous month, teachers were asked to provide the names of students they considered to be "good," "average," and "poor" mathematics students. Each teacher provided the names of two students in each category. Daily class schedules were pulled for each of these students and a memo was sent to each asking him/her to participate in a focus group interview dealing with mathematics education. Students were asked to indicate which period would be most convenient for them to dedicate to the focus group interview. All but three of the nominated students indicated that they would participate. Due to time and schedule constraints, eighteen of the original 72 nominees (25%) participated in the interviews. These students met in "mixed-ability" groups and student membership in each of the three groups "good," "average," and "poor" is described in Table 3 below. "Poor" students are numerically underrepresented because two students invited to participate were absent from school on the scheduled interview data. The teachers of the other two "poor" students did not allow them to leave their class in order to participate in the interview.

Table 3 focus group participant characteristics

	"Good"	"Average"	"Poor"
Number of students	7	8	3

Focus group participants were told that their teachers had nominated them to participate in focus group interviews dealing with math education. Several students were concerned that they had been asked to participate because they were having trouble with math, all students were told that they should consider themselves "expert consultants" to the school since those responsible for organizing the focus group interviews were primarily interested in how to *improve* the state of mathematics education at Mar Vista High School.

Students were asked guiding questions and were allowed to pursue other related topics as they arose. They were encouraged to talk with each other and not necessarily to the interviewer. All four groups of students participated in the interviews, which lasted approximately fifty minutes. A sample of guiding questions is provided below:

- Why is math a requirement in high school?
- What is "mathematics?"
- How are "good" math teachers different from "bad" math teachers?
- What does an "A" in mathematics mean?
- What kind of interaction goes on in your math classroom?
- Why do some students fail mathematics?
- Why do some students do well in mathematics?

- How do you feel about assessments in math?
- How does math compare to other subjects?

Upon completion of the four focus group interviews, audiotapes were transcribed and thematic analysis was performed. Notes from the interviews and interview transcriptions were analyzed in terms of student responses to the guiding questions listed above. Representative phrases and expressions were kept intact while recurrent ideas were paraphrased. Student responses to specific guiding questions were pooled into specific categories based on the questions they addressed.

Investigation #4

Investigation #4 is designed to provide responses to research questions 1, 2, 4, 5, and 7. It relies upon statistical analysis of student transcripts and anecdotal links between transcripts and interview comments from Investigation #3. Transcripts are dated 6 February 1998.

In Investigation #2, each member of the Mar Vista High School mathematics department was asked to provide the names of two "good" mathematics students, two "average" mathematics students, and two "poor" mathematics students. One teacher provided names after transcripts for the other students had been pulled, two names did not appear among registered students, one student had transferred to another school and one student had withdrawn to the Mar Vista High School learning center. Therefore, transcripts for 56 of these 72 students were obtained and a database was created from the following items for each of these 56 students:

- teacher rating ("good," "average" or "poor" math student)
- gender
- grade
- ethnicity
- residence zip code
- weighted scholarship GPA
- class rank
- citizenship GPA
- earned credits as a percentage of attempted credits
- most recent Stanford Reading test score
- most recent Stanford Mathematics test score
- AB65 writing sample ("mastered" or "not mastered")

Several statistical analyses (correlations, regression, factor analysis, and analysis of variance) were performed using this database. Additionally, this investigation explores

relationships between the transcripts of four individual students who participated in focus group interviews and their comments in that setting

Chapter Four: Results

The study responds to the following questions:

- 1 What factors contribute to a student's success in high school mathematics?
- 2 How do various groups of students experience high school mathematics differently?
- 3 How are slogans and ideology employed by high school mathematics education?
- 4 How does tracking reflect the hegemonic role of high school mathematics education?
- 5 How is high school mathematics characterized by student resistance?
- 6 How does interaction/dialogue in the mathematics classroom affect student confidence in high school mathematics?
- 7 How does high school mathematics function to "objectify" students and teachers?

The following twenty statements summarize the findings of the study in response to the research questions.

- 1 Mathematical confidence has a strong causal effect on a student's academic grade in his/her mathematics class.
- 2 A student's placement in a college-prep or non-college-prep mathematics class has a causal relationship with his/her mathematical confidence. Teacher rating of student math ability depends to a large degree on whether or not the student is successfully enrolled in a college-preparatory math course.

- 3 The degree of interaction a student experiences in his/her mathematics class has a causal relationship with his/her mathematical confidence. Interaction and dialogue are important factors not so much for their "cognitive" merits, but for their implications as the means for students to understand and gain control over their environments and others within school settings.
- 4 Mathematics teachers generally believe that a "good mathematics student" is one who does his/her work. A student's overall classroom behavior is a significant factor in his or her success in high school mathematics.
- 5 Teachers associate effort with the success of "good mathematics students." They associate a lack of effort and deficient intellect with the failure of "poor mathematics students."
- 6 Students reject the notion that someone who receives an academic grade of "A" knows or understands mathematics better than a student who does not receive an "A."
- 7 Students and teachers seem to accept the premise that students *choose* whether or not they will be "successful" in school mathematics classes.
- 8 Students link success in mathematics to certain teacher characteristics.

- 9 Successful mathematics students learn to do what is required to get the grade they desire
- 10 Many students describe mathematics as a "thing" that is "thrown at" students and which usually "bounces off" them
- 11 Most students describe mathematics as "the steps you need to use of find the answer "
- 12 A student's primary language, writing ability, mainstream status, and general educational level are significant factors in a student's success in high school mathematics courses. This has serious implications for the mathematics education of students from subordinated ethnic or linguistic groups in an era when specialized math courses for these students are under fire.
- 13 Student resistance to mathematics is highest at the most advanced levels of mathematics and lowest at the other end of the school mathematics spectrum.
- 14 Girls continue to suffer more than boys in school math classes. Although they maintain higher average math grades than boys, they are significantly less likely to seek careers as mathematicians or scientists than their male peers.

15 Students express a kind of "blind faith" in the construct of what they consider to be abstract mathematics, much as people adhere to various mythologies. Teachers maintain hegemonic classroom relationships by exploiting these beliefs.

16 Ideology and slogans play active roles in mathematics education. Ideologically, teachers and other educational actors consider a student's success in mathematics to be indicative of that student's overall academic potential.

17 Tracking serves a hegemonic function in traditional school systems, especially within the particular context of mathematics education.

18 A significant factor influencing the degree of resistance students display toward mathematics is their belief in their ability to succeed in the system of school mathematics, their likelihood of "conquering" the "enemy" of mathematics.

19 Successful players in the school mathematics game view the subject as a necessary obstacle, at best a means to ends that appear vague and undefined.

20 Since teachers produce by far the most language in traditional mathematics classrooms, what, why, and how they say what they do to students has either positive or negative consequences in terms of student success in mathematics.

Research Question #1: What factors contribute to a student's success in high school mathematics?

If a student's academic grade in mathematics is indicative of his/her success in the subject, then several factors contribute significantly to success in high school mathematics. Investigation #1 relies upon path analysis and regression to respond to this first research question from an affective point of view. Path analysis of the pre-diagram provided in the "Methodology" section of this paper produced the post-analysis diagram in Figure 3.

The final path diagram suggests that **a student's grade (9-12) and placement in a college-preparatory math course both have significant causal effects on his/her resistance to mathematics, value of mathematics, degree of interaction, and objective view of mathematics.** These four composite variables, in turn, exert an effect on the student's mathematical confidence. **Mathematical confidence, according to the model, has a strong causal effect on a student's academic grade in mathematics class.** The model also suggests that a student's mathematical confidence has a causal effect upon a student's plans to become a mathematician or a scientist. Finally, the model suggests that a student's gender has a significant causal effect upon one's mathematics grade.

Decomposition of bivariate covariation for all pairs of variables related to academic grade is shown in Table 5 below

Table 5: Decomposition of Bivariate Covariation with Academic Grade

	gpa/ track	gpa/ gender	gpa/ grade	gpa/ resist	gpa/ value	gpa/ object	gpa/ interact	gpa/ confidence
original covariation	.110	.055	-.074	-.349	.204	.000	.115	.417
causal - direct	.077	.066	0	0	-.15	0	0	.410
causal-indirect	.012	0	-.0012	-.13	.234	.014	.059	0
total causal	.089	.066	-.0012	-.13	.084	.014	.059	.410
noncausal	.021	-.011	-.073	-.219	.12	-.014	.056	.007

According to the post-analysis path diagram (Figure 3), the variables that share the strongest causal relationship with a student's academic grade in mathematics class are **mathematical confidence** (total causal = .410), **placement on a college-preparatory/non-college-prep mathematics track** (total direct causal = .089) and **value** (total causal = .084). Therefore, these three factors may be considered to be significant factors that contribute to a student's success in high school mathematics.

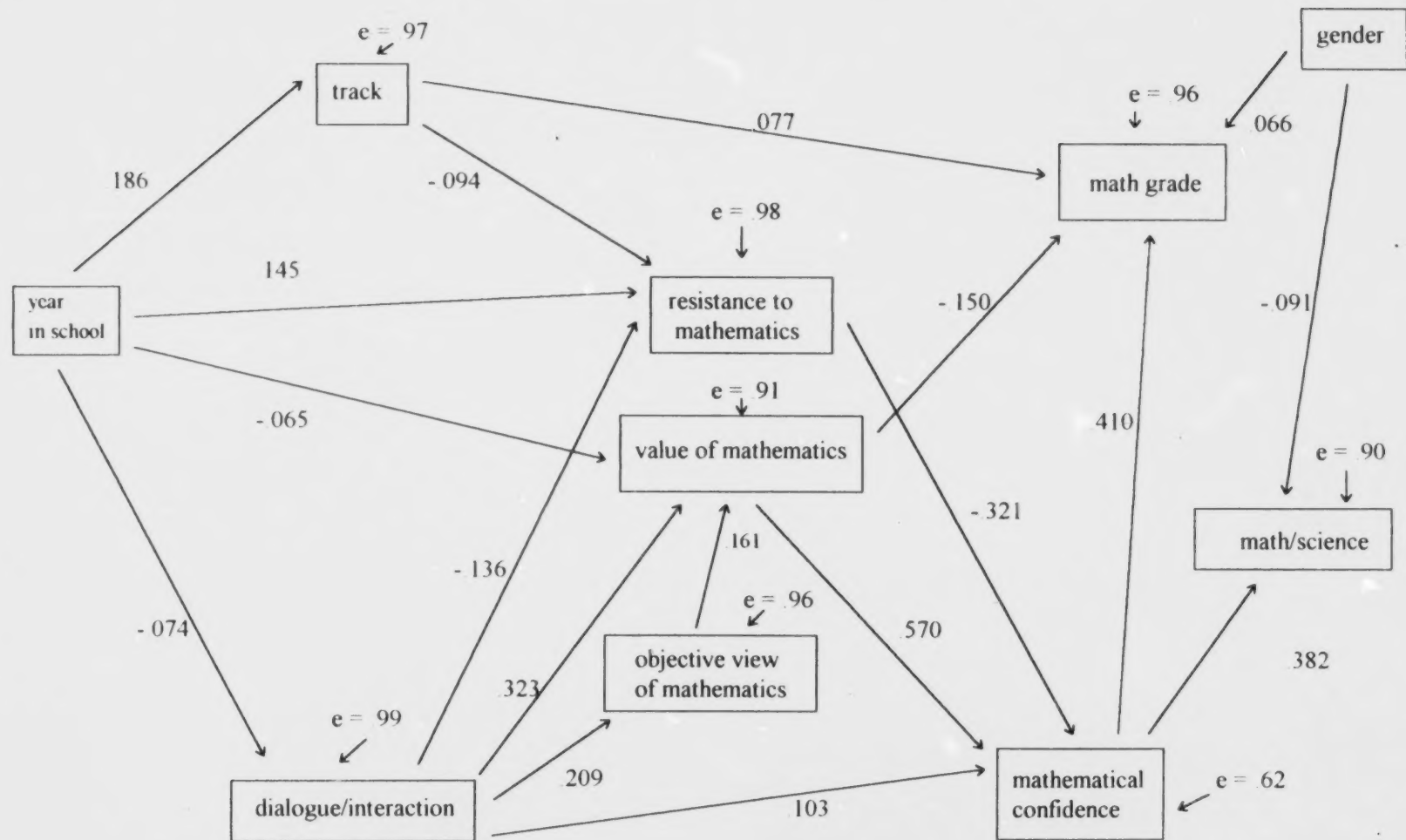
A corollary question might be to ask what factors contribute most to a student's "mathematical confidence." Decomposition of bivariate covariation for all pairs of variables related to "mathematical confidence" is shown in Table 6 below:

Table 6: Decomposition of Bivariate Covariation with "Confidence"

	confid/ interact	confid/ object	confid/ resist	confid/ value	confid/ track	confid/ grade
original covariation	.367	.161	-.559	.728	.028	-.093
causal - direct	.103	0	-.321	.570	0	0
causal-indirect	.248	.092	0	0	.030	-.026
total causal	.351	.092	-.321	.570	.030	-.026
noncausal	.016	.069	-.238	.158	.002	-.067

According to the model described by the post-analysis diagram (Figure 3), the variables which share the strongest causal relationship with "mathematical confidence" are value (total causal = .570) and interaction (total causal = .351). **This suggests that a student's placement in a college-prep or non-college-prep mathematics class has a causal relationship with his/her mathematical confidence.** It also suggests that the degree of interaction a student experiences in his/her mathematics class also has a causal relationship with his/her mathematical confidence.

Figure 3: Post-Analysis Diagram



Decomposition of bivariate covariation among selected other variables in the diagram suggests other conclusions. Table 7 provides this information:

Table 7: Decomposition of Bivariate Covariation among Select Variables

	resist/ interact	resist/ grade	value/ grade	value/ interact	value/ object	mthsci/ confid
original covariation	-.073	.148	-.101	.371	.235	.396
causal - direct	-.094	.145	-.065	.323	.161	.382
causal-indirect	0	-.017	-.026	.034	0	0
total causal	-.094	.128	-.091	.357	.161	.382
noncausal	.021	.020	.010	.014	.074	.014

As evidenced by Table 6, the final path diagram supports an assumed causal relationship between "math/science" (item 30: I will become a mathematician or a scientist) and "mathematical confidence" (total causal = .382), between "student value of mathematics" and "interaction" (total causal = .357), and between "student value of mathematics" and "object" (total causal = .161). It is interesting to note the **causal relationship supported by the model between "student resistance to mathematics" and grade** (total causal = .128). Inverse causal relationships are supported between the following pairs of variables: "resistance to mathematics" and "interaction" (total causal = -.094) and "student value of mathematics" and grade (total causal = -.091).

Regressions suggest that additional factors contribute to student success in high school mathematics. Stepwise regression using all survey variables as independent variables, "academic grade" as the dependent variable ($p < .05$) and substitution of mean

values for missing data produced the following equation $y^{\wedge} = .383(\text{"good," item 2}) + .080(\text{"gender"}) + .121(\text{"teacher perception," item 17}) - .082(\text{"cant," item 20}) - .101(\text{"hate," item 3}) + .057(\text{"afraid," item 29})$. For this equation, $R = .527$. Alone, the bivariate correlation between "academic grade" and "good" (item 2) produced $r = .509$ and the correlation between "academic grade" and "mathematical confidence" produced $r = .417$. The regression equation using "good" (item 2) as the dependent variable ($p < .05$) had $R = .613$ and included the following variables: academic grade with a weight of .349, "favorite" (item 11) with a weight of .182, "teacher perception" (item 17) with a weight of .149, "forward" (item 10) with a weight of .134, and "why" (item 1) with a weight of .120.

Investigation #2 responds to the first research question from the teacher's point of view. **Nine of the eleven interviewed teachers indicated that a "good mathematics student" is one who "does his or her work."** Definitions of "good mathematics students" could be grouped into eight categories: 1) attendance-related comments, 2) responsibility-related comments, 3) Homework/effort-related comments, 4) attitude-related comments, 5) intellect-related comments, 6) grade-related comments, 7) interaction-related comments, and 8) organization-related comments.

The frequencies of teachers' comments when asked to provide the characteristics of "good mathematics students" are listed in terms of the above groups in Table 8 below

Table 8. "Good students" - Comment Frequencies by Category

	attend	respons	HW/effort	attitude	intellect	grade	interact	organized
frequency	2	3	10	7	4	4	4	5

The frequencies of teachers' comments when asked to provide the characteristics of "poor mathematics students" are listed in terms of the above groups in Table 9 below.

Table 9 "Poor students" - Comment Frequencies by Category

	attend	respons	HW/effort	attitude	intellect	grade	interact	organized
frequency	3	2	12	3	9	5	4	3

Teacher expectations of student mathematics ability appear more linked to behavioral indicators than cognitive ones. Structured interviews with teachers in Investigation #2 allowed individuals to provide the names of "good," "average," and "poor" math students while also defining characteristics that produced these groupings. Data obtained from Investigation #2 suggest **that a major factor related to student success in high school mathematics is whether or not students do their homework and exhibit effort in doing so.** As previously noted, "effort" seems to manifest itself in mathematics classrooms primarily in the degree to which students complete and hand in regularly-assigned homework. Attitudinal factors ("the student never makes excuses," "the student is polite," "the student has a studious attitude," "the student adds to a positive atmosphere") are also significant factors in teachers' perceptions of "good mathematics students." Teachers provided more comments regarding the intellectual characteristics of their "poor mathematics students" ("they have a difficult time handling the material," "he's bright enough but lacks maturity," "he's just losing it right now and can't keep up," "she lacks the ability to discern patterns") than they did when describing their "good mathematics students." It therefore appears that **students' attitudes are considered by teachers to be significant factors in students' mathematical success**

and that students' intellects are considered to be significant factors in students' lack of mathematical success.

What is perhaps most interesting about Investigation #2 is the way "good students" and "poor students" are characterized by teachers. "Homework completion/effort" was named 10 times by teachers describing characteristics of "good" math students. It was named 12 times by teachers describing "poor" math students. Evidently, "effort" is considered central to both mathematical success and mathematical failure. What is surprising, however, is the low frequency of comments referring to intellectual qualities of students. Only four remarks were made with regard to "good students." Teachers made more comments related to intellect (9) when describing the deficits of poor students. One might conclude that a "good" math student, then, is in no way recognized as brilliant, creative, or incisive, rather, he or she displays effort and a positive attitude, maintaining an organized notebook. Poor students lack work habits but their teachers also justify their ratings with perceptions of intellectual shortcomings. These findings suggest tremendous arrogance on the part of math teachers. Good students will never be brilliant enough, poor ones are both lazy and dumb.

Focus group interviews with students suggest that mathematics education is less about mastery of abstract concepts than it is about willingness to participate and display appropriate behaviors. Investigation #3 provides responses to the first research question from the students' points of view. All focus groups of students provided the same response when asked, "What does an 'A' in mathematics mean?" All students strongly rejected the notion that a student who receives an academic grade of "A" knows or

understands mathematics better than a student who does not receive an "A." According to one student, "An A doesn't mean that a person has knowledge of math. It just means that that do their work." Another student said, "Some students with A's get things done fast. They don't necessarily understand math better." Many students used the word "choose" when they responded to questions related to mathematical success. For example, one student said that people who are "smarter" don't have to do as much work because they "know the stuff." These people, according to one student, often "choose not to do work." Another student suggested that "some student choose not to do the work because they have other classes that are important or more interesting to them."

Investigation #3 asserts the idea that **students actively choose whether or not they will be "successful" in school mathematics classes.**

When asked the question, "Why do some students do well in math?" at least one student in each focus group stated that these successful students do their homework. According to one student, "People who do well in math do work and turn it in. That's not the same as knowing the work." Others said that successful students pay attention in class, take notes, and ask questions. Grades seem to be one reason why students try to do well in mathematics classes. One student said,

Take me, for example. I think history is much more interesting than math, but I'm getting a better grade in math. My grades are in reverse to my interests. That's not unusual. Grades are just a part of the system. You've got to try hard in all your classes because all your grades will matter.

Students provided a variety of reasons why some students choose to be successful in school math and why some choose to fail. Many of these reasons comprise what could be considered a "mythology" of school mathematics. Some students indicated that they want to know math so they can get good jobs in the future. Some said successful students had "seen the stuff before, in schools in Mexico" before they encountered "new" principles in their mathematics classes. Several students mentioned parental expectations as reasons why students are successful in mathematics. One commented that "my mom and dad expect me to get a good grade, but they can't help me with the math."

Every focus group mentioned a direct connection between students' success in mathematics and teacher characteristics. According to the students, teacher characteristics that lie behind student success in high school mathematics (according to the students) are 1) a willingness to work one-on-one with students, 2) a willingness to "make sure you know it," teacher effort to ensure that all students learn the mathematical material, and 3) the ability to clearly explain and to "keep their students in check" (maintain adequate discipline in the classroom).

Conversely, students in focus groups made numerous comments related to student failure in mathematics. **No student suggested that mathematical failure is due to students being "stupid" or unable to grasp the mathematical content required by math classes.** Many comments dealt with students' lives outside of school and the reasons why homework doesn't get done. According to one student, there are people who "get it" but don't have time to get their homework done. "Some students don't have time to go through all the steps you need to go through to get the right answer to a

problem," explained one. Another student described time conflicts between work and school. Two students said that they don't bring their homework home because there's nobody at home who can help them to get it done. The nature of math homework also was mentioned. "Some people encounter difficulty and then they skip the unpleasant problems," explained one student. "Homework can be valuable, but most of the time it just requires students to do the same things over and over," said another. "Some students are forced to skip over math they should have learned in the past. Math is cumulative, and since people miss some of their past maths, they are doomed."

Student choices and the consideration afforded them by teachers were named as reasons for student failure in mathematics, just as they were named as factors in student success. One student said, "I do all my other homework! I even bring my math book home every day! I just never open it! I don't know why!" The same students added that, "Some math teachers are really blunt. They think that just because they're teaching math, they don't have to be polite to people," said one student. "Teachers need to remember that even though they've seen the stuff a thousand times, it's students' first time seeing the math. Students commented on grading policies and lesson organization: "sometimes math lessons look like a list of notes that are never explained." Many students suggested that one way to help students to become more successful in math would be to allow them to choose their math teachers. "Teachers who teach just one way will have a lot of F's. Students learn in different ways, so teachers should be able to teach in different ways." Most of the time, said one student, "teachers don't realize that the things they say just 'bounce off' their students." Along these lines, some teachers "treat math like so many

grains of sand that they try to keep from draining out the other side of your head once they put them in." **Three students mentioned that many math teachers talk at students rather than talking to them.** Interestingly, one student made the mathematically-appropriate suggestion that teachers "change their ratio of talk to explanation."

Investigation #3, therefore, suggests that a variety of factors contribute to student success in school mathematics. **The most significant factor appears to be a student's willingness to work hard to complete assigned homework.** Closely linked to this factor is the motivation provided by assignment of an academic grade. Additionally, successful mathematics students, in the words of one focus group member, "adapt to their teacher's style." **Successful mathematics students learn to do what is required to get the grade they desire. Teacher characteristics are significant factors which contribute to a student's success or failure in mathematics.** More student comments fell into this third category than into any of the others previously mentioned.

Transcript analysis considered way in which several systemic variables function together to define the "academic status" of individual students. Investigation #4 addresses research questions from a systemic perspective. If student success in mathematics is tied to teacher perception of student mathematics ability, then teacher nominations from Investigation #2 may be used to reflect one measure of student success in mathematics. Although "teacher rating" ("good," "average," or "poor") shared a Pearson correlation of .431 with students' standardized math test scores, other variables shared stronger correlations with "teacher rating." The bivariate correlation between "teacher rating"

("good," "average" or "poor") and a student's overall *citizenship* gpa (not based on academic grades, but more on classroom behavior or other considerations) was significant at $p < .001$ with $r = .688$. This suggests that **a student's overall classroom behavior is a factor in his or her success in high school mathematics**. Citizenship GPA rarely depends on students' demonstrated mastery of course content, it depends much more upon how students adhere to classroom norms like being on time, raising one's hand prior to speaking, refraining from disrupting the class or a teacher's lectures, and demonstrating behavior that the teacher expects or has identified as "appropriate."

Additionally, the correlation between "teacher rating" and "writing sample" (an English-language writing sample required to be mastered prior to graduation) was significant ($p < .001$) with $r = .450$. This may suggest that a student's communication skills, writing ability, mainstream status, or general educational level are significant factors in a student's success in high school mathematics courses. This finding has serious implications for the mathematics education of students from subordinated ethnic or linguistic groups in an era when specialized math courses for these students are under fire. If math teachers judge students based on English-language proficiency, non-standard English speakers are immediately handicapped, especially within a system where teacher expectations for silence and non-disruptive behavior virtually prohibit these students from developing English fluency as well as from gaining an understanding of mathematics course content. However, this finding may simply assert that success in mathematics is linked to success in other subjects (including language arts). Students who do well in general are likely to do well in mathematics courses because they have acquired study

habits and writing abilities that are prerequisite for success in mathematics and all academic subjects

Finally, the correlation between "teacher rating" and a student's overall academic grade point average when controlled for grade level was significant ($p < .001$) with $r = .708$. It appears that a student's overall academic success in various high school classes over time is closely related to his or her teacher's perception of that student as a "good" math student, as an "average" math student or as a "poor" math student. Of course, observant and experienced teachers must be able to identify traits which are likely to predict student success in school. Again, success in mathematics may be representative of success in school in general. A close link between teacher rating and overall GPA suggests that the criteria used to evaluate student achievement is, at least, consistent.

One of the most important findings from Investigation #4 is that only students who successfully "play the game" of school math will win. Those who either do not "know the rules" (i.e. in terms of language or behavioral norms) or those who refuse to play by those rules will lose. Success in school mathematics, especially since this construct is defined by math teachers, depends on the ways in which students are able and willing to play the game in which they find themselves contestants.

Research Question #2: How do various groups of students experience high school mathematics differently?

Investigation #1 relies on statistical analyses to describe the ways in which students appear to experience and perceive high school mathematics and their relationship to the subject. Various groups of students appear to experience high school mathematics quite differently in a number of ways. Students in more advanced levels of college-preparatory mathematics feel they are allowed to participate more in learning the subject than those in lower levels of school mathematics. More advanced students have characteristics of "system-players" who are winning the game of high school mathematics. **They are, however, winners whose significantly high level of resistance to the subject reflects tremendous dissatisfaction with the game itself.** The primary analysis used to describe these differences is analysis of variance.

The first finding relevant to the second research question is that a variety of key variables differ significantly among levels of mathematics courses. Table 10 describes significant differences among these variables.

Table 10: Highly Significant F Ratios for Selected Items
(all significant at $p < .001$)

Variable	F
"My teacher asks me to explain answers" (item 15)	5.07
"I am good at math" (item 2)	7.55
"I feel helpless in my math class" (item 24)	3.66
academic grade	4.61
"I will become a mathematician or a scientist" (item 30)	3.70
"I am expected to stay quiet in my math class" (item 9)	5.26
"I know what is required to earn an A in math" (item 26)	3.81
resistance to mathematics	3.53

Table 11 provides the between group means for each key variable for groups in homogeneous subsets

Table 11: Means For Student Responses To Selected Items By Math Course
(the number of students in each subset follows the mean of each item)

	Math A	Math B	Course 1	Course 2	Course 3	PreCalc	Calculus
"Explain"	3.82(94)	3.39(44)	3.78(350)	3.55(280)	3.42(195)	3.30(47)	3.21(22)
"Good"	3.05(94)	2.84(44)	3.43(353)	3.28(283)	2.28(196)	3.79(47)	4.34(23)
"Helpless"	2.29(91)	2.84(44)	2.14(351)	2.27(282)	2.46(194)	2.21(47)	1.78(23)
GPA	2.28(87)	2.11(37)	2.58(332)	2.48(267)	2.61(185)	2.96(45)	3.30(23)
"MathSci"	1.86(93)	1.59(44)	1.87(350)	2.01(282)	2.11(197)	2.64(47)	2.13(23)
"Quiet"	4.01(92)	3.65(43)	3.87(351)	3.48(277)	3.68(194)	3.09(47)	3.43(23)
"ReqA"	4.25(91)	4.34(44)	4.48(351)	4.48(282)	4.51(196)	4.55(47)	4.57(23)
Resistance	2.35(85)	2.60(40)	2.20(323)	2.29(263)	2.29(184)	2.20(44)	2.13(22)

The results of this first analysis of variance suggest several responses to the second research question. Students in more advanced levels of math indicate that they are "asked by their teacher to explain answers to math questions" less than students in less advanced levels of math. Students in higher levels of math also seem to respond more affirmatively to the statement "I am good at math" than do students in lower levels of mathematics. **Students in the highest two levels of mathematics indicate that they feel less helpless than do students in other levels of mathematics**, though the means are not ordered by math course level. Students in higher levels of mathematics receive higher academic grades, students in math B receive the lowest grades.

Table 11 provides insight into a similar trend when student responses to item 30 ("I will become a mathematician or a scientist") are examined. Student responses become increasingly affirmative as they advance up the hierarchy of math courses. Students in lower levels of mathematics indicate that they are expected to stay quiet in their math

classes more than do student in higher levels of math. Students in higher levels of mathematics indicate that they "know what is required to earn an A in math" more than do students in lower levels of mathematics. Finally, an interesting finding is **that student resistance to mathematics is highest at the most advanced levels of mathematics and lowest at the other end of the mathematics spectrum.**

Another analysis of variance revealed significant differentiation within key variables in terms of grade levels. These key variables are displayed in Tables 12 and 13.

Table 12. Between Groups Variance - by Grade Level

Variable	F
"I feel afraid when I'm in my math class "	6.44**
"Math has nothing to do with my life "	4.16*
"I feel helpless in my math class "	9.27**
"My teacher asks me to explain answers."	14.84**
"I am good at math."	5.17**
math value	5.15*
"I am expected to stay quiet in my math class "	6.19**
resistance	7.90**
"Some people can't learn to do math."	5.28**
"I expect to use math in my life when I am an adult."	7.10**

* $p < .005$

** $p < .001$

Table 13. Means For Student Responses to Selected Items by Grade Level
(the number of students in each subset follows the mean of each item)

	Grade 9	Grade 10	Grade 11	Grade 12
"Afraid"	1.76(357)	1.83(305)	2.07(241)	2.15(192)
"Irrel"	2.13(359)	2.21(310)	2.34(243)	2.52(192)
"Helpless"	2.07(358)	2.23(312)	2.55(240)	2.54(191)
"Explain"	3.85(361)	3.64(310)	3.25(243)	3.29(189)
"Good"	3.48(363)	3.26(313)	3.15(244)	3.25(192)
Math value	4.00(350)	3.83(308)	3.86(236)	3.78(190)
"Quiet"	3.83(359)	3.67(307)	3.67(242)	3.53(192)
Resistance	2.15(338)	2.29(288)	2.41(222)	2.47(184)
"Somecan't"	2.54(360)	2.70(308)	2.95(241)	2.87(188)
"Useadult"	3.82(360)	3.77(312)	3.67(242)	3.75(191)

This analysis suggests that student responses to certain items varies significantly by grade level. Table 13 provides student responses to selected items by grade level. Underclassmen said they are less "afraid" than the upperclassmen. It is interesting to note that when asked to respond to the statement "Math has nothing to do with my life," underclassmen agreed less than do upperclassmen. Underclassmen indicated that they feel less "helpless" than do the upperclassmen. Upperclassmen indicated that they are asked to explain their answers to math problems less than the underclassmen say they are.

Upperclassmen responded more negatively to the statement "I am good at math" than did the underclassmen. Seniors had a significantly different mean for "math value" than the freshmen, and upperclassmen indicated that they were expected to be quiet in their math classes less than the underclassmen did. It is interesting to note that underclassmen had lower values for "resistance to mathematics" than did the upperclassmen. Freshmen and seniors were farthest apart in terms of this variable. When faced with the statement "Some people can't do math," the upperclassmen agreed significantly more than did the underclassmen. Finally, upperclassmen indicated that they "expect to use math as adults" less than did the freshmen.

Investigation #1 supports two major assertions. First, the more students learn about the "game of mathematics," the less they enjoy playing. The second is that mathematics becomes increasingly irrelevant to students the longer they study the subject. Why do older students demonstrate less "faith" in school mathematics than younger students who have spent less time "playing the game?" Instead of empowering students to

find ways in which mathematics is beautiful or useful in improving life or solving problems, school mathematics appears like a training camp in which students are forced to work out without any sense that their skills will benefit them in "real life," life outside of school. This has disturbing implications for the public school system, where mathematics is almost universally revered as the most sacred and untouchable of academic subjects. Investigation #1 suggests that the very *existence* of school mathematics, not just its characteristics, be questioned.

As a vast body of educational research documents, girls continue to suffer more than boys in school math classes. Using gender as the differentiating factor provided more responses to the second research question. Table 14 shows the group means for variables that have significant differences:

Table 14: Between Groups Variance - Boys vs. Girls

Variable	x (female)	x (male)	F
"I look forward to doing math." (item 10)	2.68	2.85	3.37**
"I am good at math." (item 2)	3.18	3.42	3.82***
"I often work with others." (item 21)	3.52	3.28	2.44*
"I can't do math." (item 20)	2.19	1.94	2.24*
"I am expected to stay quiet in class." (item 9)	3.60	3.71	2.31*
"I feel helpless in my math class." (item 24)	2.39	2.19	2.15*
"I will become a mathematician/scientist." (item 30)	1.82	2.12	5.03****
academic grade	2.57	2.49	2.78**

* $p < .10$

** $p < .05$

*** $p < .01$

**** $p < .005$

Girls indicated that they "look forward to working on math problems" less than boys do; they also tended to disagree with the statement, "I am good at math" more than boys did. Girls stated that they "often work with other students in groups to do math"

more than boys did and it is interesting to note that girls said that they "are expected to stay quiet" in their math classes *less* than boys did. Although girls reported significantly higher math grades than boys, they also responded more affirmatively to the statements "I can't do math" and "I feel helpless in my math class" than their male peers. Finally, girls indicated that they were far less likely to agree with the statement, "I will become a mathematician or a scientist" than were boys.

It is disturbing to find that girls express such low confidence in their own mathematical abilities despite earning significantly higher academic grades than boys do in math classes. Girls do not blame the subject or their teachers for their apparent shortcomings; **they blame themselves** when they indicate that they "can't do math" and that they are not "good in math." Although girls responded that they do not feel they are expected to stay as quiet in math classes as boys did, they also indicated that they look forward to math much less than boys do. Consequently, girls indicated a significantly lower degree of interest in becoming mathematicians or scientists. Clearly, teacher behaviors affect the degree of attention girls receive, the configuration of student activities (noting that girls apparently prefer collaborative work more than boys do), and expectations for student participation, questions, and other interaction.

Investigation #2, using structured interviews, relied on teacher input to identify characteristics of "good" math students, "average" math students and "poor" math students. Table 15 below provides the number of students from each math class that appeared as "good," "average" and "poor" students according to teachers' classification.

Table 15 Classification by Math Course

	Math A/B	Crs 1 repeat	Course 1	Crs 2 repeat	Course 2	Crs 3 repeat	Course 3	Pre- Calc	Stats
N good	2	1	6		2		2	2	
N average	2		9	1	7	1	2		2
N poor	10	3	4	2	2	1			

Table 16 classifies teacher ratings of students in terms of whether or not the student's math class is college-preparatory or non-college-preparatory:

Table 16 Classification by Track and Repeat Status

	Non-CP	non-repeat College-prep	repeat College-prep
N good	2	12	1
N average	2	20	2
N poor	10	6	6

These results suggest that teacher rating of student mathematics ability depends to a large degree on whether or not the student is successfully enrolled in a college-preparatory math course. Students in non-repeat college-preparatory math courses are more likely to be considered "good" math students than those in either non-college-prep courses or repeat courses. That teachers were able and willing to classify students as "good," "average" and "poor" math students suggests that groups of students do experience math differently (from the teacher's perspective). As described in the responses to research question #1, teachers also clearly differentiated between students who exhibit desirable attributes (doing homework, being polite, "trying hard") and those who exhibit undesirable attributes (being "rude," lacking motivation, not doing work). Investigation #2 suggests that teachers view groups of students in each of their classes

very differently. It follows that students will therefore have different experiences in their mathematics classrooms.

In light of findings in response to Research Question 1, varied teacher ratings of students in different tracks implies that students are virtually unable to alter their perceived abilities as mathematical thinkers. Teachers, to a large degree, determine whether or not a student will successfully exit a course with the grade required for entrance to a successive math class. Teachers determine the criteria by which a student is enabled to transition within the college-prep sequence of math courses. In effect, "non-college-prep" students are labeled by teachers and others responsible for course placement in schools as "poor" math students, an identity these students appear to internalize. This in part explains why their responses to survey items in Investigation #1 were so different from the responses of their "college-prep" peers.

Investigation #3, using focus group interviews, asked students to differentiate between students who fail mathematics and students who do well in math classes. Focus group members described several different types of students. Several described students whose life outside of school conflicts with the objectives and/or expectations of their math class. Some suggested that the family life of these students contributes to their failure; one young woman explained how her parents stopped taking math many years ago and are therefore unable to help her with her math homework. Several focus group members suggested that many students face time constraints due to family or work responsibilities.

Focus group members also described characteristics that make certain students successful in high school mathematics. These students "take notes," "pay attention in

class," and "do their homework and turn it in." According to one focus group member, these students are able to "adapt to a teacher's style." Some, but not all, of these students are also "motivated by a grade," "want to know math so they can get good jobs in the future," or are expected to do well by their parents. One focus group member explained that some students "have seen the stuff before, in schools in Mexico," before they encounter new material here in U.S. math classes. Generally, students may be classified as those who cooperate and try to do what is expected and those who do not. Focus group members suggested that the first group often consists of students who have some reason to try and do well in math. These reasons vary but they provide students with some incentive to work.

Investigation #4, using transcript analysis, relied primarily on analysis of variance and correlations to respond to research question 2. **Transcript analysis suggests that students experience mathematics differently based on where they live.** In this investigation, zip code was used to represent the distance between a student's residence address and the school. Additionally, zip code can be used as a proxy for the socioeconomic status (SES) of students attending Mar Vista High School. Two communities of particular interest in this investigation are Imperial Beach and San Ysidro. Located near the Pacific Ocean, Imperial Beach has higher real estate values than San Ysidro, located just north of the U.S./Mexico International border. Although most residents in both communities occupy rental property, zip code comparison allows for ordinal classification of students' SES levels. Table 17 displays variables which differed significantly when zip code was used as the differentiating variable. "Teacher rating"

ranges from 0 ("poor" math student") to 2 ("good" math student). "Writing sample" ranges from 0 (not mastered) to 1 (mastered).

Table 17: Significantly Different Means for Teacher Rating by Community

variable	x Imperial Beach	x San Ysidro	F	significance
teacher rating	1.19	.42	2.76	.051
writing sample	.70	.083	6.02	.001

Clearly, students in the two communities differ in terms of English-language proficiency and teacher rating. Students from San Ysidro are much less likely to have mastered the writing sample than students from Imperial Beach. Teachers rated students from Imperial Beach more positively than they rated students from San Ysidro. The Pearson correlation between "teacher rating" and "writing sample" ($p < .000$) had $r = .45$. The correlation between "teacher rating" and "citizenship GPA" ($p < .000$) had $r = .69$. The correlation between "teacher rating" and "overall GPA" (including all subjects taken in high school) when controlled for grade level ($p < .000$) had $r = .708$. It is clear that a teacher's perception of students as "good," "average," or "poor" math students has a relationship to students' overall academic records. Additionally, one may conclude that teachers' ratings of students as "good" math students, "average" math students, or "poor" math students has a relationship to the students' writing sample scores and citizenship grades. This suggests that a student's address, ability to pass the writing sample, and behavior in class contribute to how that student may experience high school mathematics.

Again, results from Investigation #4 support the assertion that some students are much more able to play the "game" of school mathematics than others. Some of the most significant reasons are outside of students' control. Data from investigation #4 suggest

that Student A, who lives close to school in Imperial Beach, has passed the writing sample, and behaves as expected in class will be more likely to be considered a "good" math student than Student B, who lives far from school in San Ysidro, has not passed the writing sample and does not behave as expected in class

None of these differentiating factors has any direct or indirect relationship to mathematical aptitude in any pure sense. Given that the writing sample is described as a one- or two-paragraph, "eighth-grade"-level measure of English fluency, one cannot even conjecture that analytical thinking is a construct gauged by the writing sample. Certainly, students from San Ysidro must be adept problem-solvers to overcome transportation, language, and expectation barriers that lie between themselves and their school. "Success" in school mathematics evidently does not depend on students' problem-solving abilities, as advocated by the California State Mathematics Framework, "mathematical power" appears mysteriously allocated to certain groups of students. How these chosen groups are identified reflects little consideration of "cognitive" mathematical skills or talents

Research Question #3: How are slogans and ideology employed by high school mathematics education?

Investigation #1 responds to research question 3 through analysis of student responses to survey items that address affective aspects of high school mathematics education. Given that student responses to each item appear on a Likert scale, a response of "1" reflects a response of "strongly disagree" whereas a response of "5" represents a response of "strongly agree." A response of "3," therefore, translates into a neutral response. Table 18 provides the means of student responses to selected survey items. Items 4, 1 and 26 relate to the perceived value of mathematics and perceived control over one's grade; items 5 and 2 relate to perceived mathematics ability; items 8 and 18 relate to the perceived utility of mathematics; items 22 and 23 relate to the objective nature of mathematics; items 25, 17, 15, 9 and 21 relate to factors affecting dynamics in the mathematics classroom.

Table 18 Means for Student Responses to Selected Items

Survey item	Survey Statement	Mean response
4	It is important to do well in math classes	4.5
1	I know why I have to take math in school	4.4
26	I know what is required to earn an A in math.	4.5
5	I was good at math in elementary school.	4.2
2	I am good at math.	3.5
8	I use math from school to solve problems in my daily life.	2.9
18	I expect to use math in my life when I am an adult.	3.8
23	Answers to math problems are either right or wrong.	3.6
22	The way to succeed in math is to memorize things.	3.6
25	My teacher believes that everybody can learn math.	4.4
17	My math teacher thinks I am a good student.	3.3
15	My teacher asks me to explain my answers to math questions.	3.5
9	I am expected to stay quiet in my math class.	3.7
21	I often work with other students in groups to do math.	3.4

Investigation #1 supports the idea that **students consider mathematics to be an important subject**. Students responded most affirmatively to items 4, 1 and 26. Students, therefore, tend to agree with the idea that it is important to do well in mathematics. Investigation #1 asserts that they know why they have to take math and that they know what to do in order to succeed ("I know what is required to earn an A in math"). Survey responses to these items would suggest that students have control over their mathematics grade. Other responses, however, suggest that students are responding to items 4, 1, and 26 the way they are expected to respond or the way in which they have been told to respond in the past.

For example, students responded more affirmatively to the statement that "I was good at math in elementary school" than they did to the statement, "I am good at math." This suggests that **students in general believe they are becoming worse at math over time**. Another related pair of items is #18 and #8. Students responded more positively to the statement "I expect to use math in my life when I am an adult" than they did to the statement "I use math from school to solve problems in my daily life." Student responses, therefore, seem to exhibit a kind of faith that mathematics will one day become useful to them (despite the self-reported drop in confidence from elementary school to high school).

Students provided slightly affirmative responses to the statements that "the way to succeed in math is to memorize things" and "answers to math problems are either right or wrong." Students provided strong affirmative responses to item #25 "My teacher believes that everyone can learn math" but only slightly affirmative responses to the items "My math teacher thinks I am a good student" and "My math teacher asks me to explain my answers to math questions." Although students provided slightly affirmative responses to the statement "I am expected to stay quiet in my math class," they also provided slightly affirmative responses to the statement, "I often work with other students in groups to do math," which would imply a verbal exchange of information and therefore some degree of noise.

In summary, **Investigation #1 suggests that ideology and slogans play active roles in mathematics education for the simple reason that numerous contradictions appear evident in student responses to survey items**. Why do students perform so poorly if they apparently know why they have to take mathematics, think it is important

and also know how to succeed? Why do students indicate that they expect to use mathematics in the future even though they indicate that they do not currently use school math in their daily lives? It is interesting to refer to previous research questions and observe that older students (grades 11 and 12) respond more negatively to item #18 (I expect to use math in my life when I am an adult) than do younger students (grades 9 and 10)

Investigation #2, using structured interviews of teachers, illuminates responses to survey items in Investigation #1. Given the strong affirmative response to item #1 (I know why I have to take math in school), focus group members were asked why they thought mathematics was a requirement in high school. Many of these responses reflect the influence of educational slogans. Although one student stated that math is "a modern form of torture," one student explained that math is "passed on by tradition. We've always had to take math. It's part of the system." Math, in this case, is considered a given, not a subject to be questioned. Many students expressed the kind of "faith in future usefulness" that appeared on the surveys: "you take [math] 'cause it becomes useful later," "it will be important when we grow up," "it will come in handy." One student said, "it's useless now, but we might use it in the future."

Other responses embody many slogans currently used to justify traditional mathematics: "it helps with comprehension," "it gets you the basics you need," "I'll need it for college and jobs - that is, depending on whether or not I get any financial aid or scholarships." Some student responses reflected a kind of cynicism: "Not even the teachers know why we take math," said one student. "It's a way to 'get the mind

running,' or so the teacher say I think that's just an excuse to get us to do it," said another.

Two of the students identified as "poor" math students by their teachers provided specific mathematical examples of what they considered "useless" mathematics. "I have no idea [why I have to take math]," said one student. "Look at this," (pulls worksheet from binder) " $2x + 4 > 6$. When am I ever going to use this?" she asked. Another "poor" student suggested that "they should stop teaching those things like, 'a over 5 plus b over 2.' Fractions aren't necessary. And all that stuff with x. I don't need to know about variables. Why do we need to learn about variables? I don't use them!"

An important finding from Investigation #2 is the **overwhelmingly pessimistic view students expressed about school mathematics and about mathematics in general**. Not one of the students admitted to being good at math, much less to liking it. This is especially troubling since most of the students forming focus groups were rated by their teachers as either "good" or "average" math students. One might assume that the "poor" students would offer the harshest critique of the subject and that "good" ones would be most positive. This was not the case. In fact, it was the "poor" students who offered the most mathematically-specific evidence in support of their criticisms. The "good" students were much more general and matter-of-fact in expressing their acceptance that math is just one part of the "school game" and just another hurdle they must leap en route to "college" or "jobs," neither of which appeared to represent a mathematically-rich dream. The "good" students were most resigned, the "poor" students most resistant. This contrast suggests that school mathematics has been perpetuated by

slogans and the ideologies of those who have successfully navigated through the subject. School math is certainly no fun for students today.

Although one student indicated that mathematics is useful in her daily life ("it helps you to avoid getting cheated out of babysitting money"), most students stated that they did not expect to use the math they were learning in school. "You'll use about half of what you learn in school," said one student. Most students said that they would not use high school math past algebra. "The only math I use now is like kindergarten math: 1 plus 1, 2 times 2, etc.," one student. One student explained that she was willing to work hard in math despite the fact that she knew that she would have to "start over" when she went to college. "I know I'm going to have to start over at the bottom when I go to college," she said. When asked why that would be necessary, she matter-of-factly responded, "that's just the way the system works. I want to have a daycare, so I need to go to college. If I want to go to college, I need to get good grades, so I have to work hard in my math class because I need that grade if I want to go to college."

When asked to provide a definition for mathematics, slogans again became evident in student responses. One student described mathematics as "basic skills." Another defined it as "common knowledge" that will "help you to compete for a job," "get references," and which "you'll need when you get older." Another student defined mathematics as "a universal language" or "a language in itself." In summary, student responses on the survey indicated that most students had a clear idea of why mathematics is required in high school. Narrative student responses, however, reveal tremendous

differences in student views and a number of recurrent slogans that characterize the current mathematics education "debate."

When asked to provide descriptors for "good" mathematics students, "average" mathematics students and "poor" mathematics students in Investigation #3, math teachers provided a response to the third research question. The most significant finding for this research question is that **math teachers seem to equate effort (in the form of homework completion) and expected classroom behavior (in the form of politeness and a "positive attitude") with a student's rating.** "Good" math students completed their homework on time, "poor" students lack motivation and a "negative" attitude. Only two math teachers included any specific cognitive observations ("he has the ability to discern patterns," and "he constantly seeks answers to fundamental mathematical questions" - both referring to boys) in their criteria and both of these observations applied to "good" math students.

Transcript analysis in Investigation #4 suggests responses to research question #3. Factor analysis (principal components) of all items used for transcript analysis (gender, grade, ethnicity, zip code, academic GPA, class rank, citizenship GPA, credits earned as a percentage of credits possible, standardized reading test score, standardized math test score, and writing sample status) suggest that a student's academic transcript contains one factor with an Eigenvalue of 4.57 and which accounts for 38.1 percent of the variance among cases. This factor is composed primarily of citizenship GPA (factor loading of .889), credits earned as a percentage of credits attempted (factor loading of .880) and overall scholarship GPA (factor loading of .930). This result responds to the third

research question since **a student's official identity within the public high school system equates to the principal factor represented by his/her transcript**. It is interesting to note that the Cronbach's alpha among citizenship GPA, credits earned, scholarship GPA and teacher rating ("good," "average," and "poor") is .872. Evidently some relationship exists between teacher perception of student mathematics caliber and a student's overall academic record (of which mathematics courses are a minor component). Ideologically, success in mathematics may be perceived to represent a student's overall academic potential. Students who are good at mathematics may be expected to excel in academics in general.

That teacher rating correlated so highly with these other transcript variables supports the idea that "good" math students are good system players. They learn what teachers and others expect of them and they deliver behaviors and products that conform to these expectations. Mathematics, revered by many as an abstract, theoretical, intellectually-challenging discipline, is the ideal context in which to observe the "formatting" power of schools. Those who experience success adhere to norms and expectations that are directly tied to the dominant ideology of the system. At least at the secondary level, success in school mathematics does not appear to be the result of demonstrated mathematical brilliance. Rather, it appears to be the result of adherence to a complex set of ideological principles that apply not only to math classrooms but to most other learning environments in traditional public schools today.

Research Question #4: How does tracking reflect the hegemonic role of high school mathematics education?

Investigation #1 provides analysis of the ways in which student responses to survey items differ depending on enrollment in a college-preparatory mathematics course. Table 19 compares college-prep and non-college-prep student responses to selected items.

Table 19: Means and F Ratios for Selected Items by Track

Variable	x (collprep)	x (non-CP)	F
"can't" (item 20)	1.97	2.41	8.05*
"memory" (item 22)	3.54	3.85	7.79***
"good" (item 2)	3.38	2.99	15.17****
"hate" (item 3)	2.66	2.99	6.45*
academic grade	2.59	2.23	12.02***
"quiet" (item 9)	3.66	3.90	4.60*
"reqA" (item 26)	4.49	4.28	5.32*
"resistance"	2.25	2.43	5.21*
"used" (item 8)	2.89	3.18	5.74*

* $p < .05$ ** $p < .01$ *** $p < .005$ **** $p < .001$

Clearly, Table 19 presents data that support the assertion that the practice of **tracking serves a hegemonic function in mathematics education**. Students in college-preparatory classes appear to feel they have greater control over their mathematics educations. This is demonstrated by significantly more affirmative responses to item 26 ("I know what is required to earn an A in math") by college-preparatory students than by non-college-preparatory students. This sense of control evidently affects students' ability to earn higher scholarship grades: college-preparatory students had significantly higher math grades than their non-college-preparatory peers.

Students in non-college-preparatory math classes indicated that they are expected to stay quiet in their math classes significantly more than students in college-preparatory classes. They also indicated that "the way to succeed in math is to memorize things" significantly more than their college-prep peers. These two responses have hegemonic implications. Students not enrolled in college-preparatory math courses seem to feel silenced and they express the view that the way to succeed is to perform rote activities (including memorization). Both of these ideas are antithetical to the "mathematical power" slogan, which emphasizes "mathematical communication" and "inquiry learning."

Other hegemonic messages can be found in the difference between college-preparatory and non-college-preparatory responses to items #2 (I am good at math), #20 (I can't do math) and #3 (I hate math). College-preparatory students responded more affirmatively to the statement "I am good at math" and more negatively to the statements "I can't do math" and "I hate math" than did the non-college-preparatory students. Hegemony, the maintenance of power relations in school society, is supported by mathematics education since data from Investigation #1 (statistical analysis) clearly describe two populations of students who differ significantly in terms of measures of mathematical confidence and affinity to the subject. If, as advanced in response to research question #1, "mathematical confidence" is a significant predictor of math grade and, as advanced in response to research question #3, math rating is significantly related to a student's overall academic record, **students who lack confidence (and its contributing factors) are doomed to remain powerless in the public school system**

Non-college-prep students expressed significantly higher values of “resistance than college-preparatory students. However, they also provided significantly more affirmative responses to the statement “I use math from school to solve problems in my daily life.” Why, one might ask, would students who apparently resist mathematics more than college-preparatory students indicate that they use math more than these apparently “empowered” mathematics students? **Counter-hegemony, in the form of resistance to mathematics, is evident when responses to survey items are compared between students on a college-prep track and students on a non-college-prep track.**

Table 20 (from responses to research question 1) illustrates the link between teacher rating of students and students’ placement in different tracks.

Table 20: Classification by Track and Repeat Status

	Non-CP	non-repeat College-prep	repeat College-prep
N good	2	12	1
N average	2	20	2
N poor	10	6	6

It is evident that **teachers consider more students from non-college-preparatory tracks to be “poor” mathematics students than to be “good” mathematics students.** Additionally, teachers consider more students from non-repeat college-preparatory students to be “good” math students than to be “poor” math students. Hegemony is evident in this data. According to school policy, students are placed in the “most appropriate” mathematics course. If this were the case, “good,” “average” and “poor” students would exist in all math courses. “Poor” students would not be clustered in non-college-preparatory courses and “good” students would not be concentrated in

college-prep courses. However, students from the non-college-preparatory track are overrepresented in the "poor" math student category, thus reinforcing the hegemonic structures that identify college-preparatory students as more likely to succeed than their non-college-prep peers

Responses by Investigation #4 (transcript analysis) to research question #3 suggest that **a student's citizenship GPA is a significant component of the major factor represented by a student's academic transcript**. The strong correlation between this factor and the teachers' rating of students as "good," "average," or "poor" relates to the data provided by Tables 8 and 9.

As previously discussed, "citizenship" grades are largely based upon students' willingness to conform to their teachers' expectations. Some expectations are explicitly conveyed to students in a variety of ways. Students who desire high citizenship grades must not question, disrupt, or criticize classroom norms or the power relationships that exist between the teacher and her students or among students in a classroom. Hegemony is enforced through both the award of letter grades (scholarship and citizenship) and via informal aspects of teacher-student dynamics. Who is selected to answer questions, when, how, and why contribute to students' understanding of both expectations and their teachers' views of individuals' roles within the systems of classroom and school. Quiet, compliant, students are awarded A's in citizenship because they appear to conform to the hegemonic structure of the traditional classroom, where the teacher controls access to both content-area knowledge and systemic privileges.

Results from Investigation #4 (transcript analysis) illuminate another aspect of the hegemonic role of mathematics education. Instead of citing characteristics that would be classified as "mathematical," teachers cited behavioral characteristics when asked to describe "good" math students. Clearly, judging students according to the degree to which they exhibit desired behaviors (completing homework, projecting a positive attitude, etc) maintains hegemony among groups in schools. Teachers benefit from this kind of structure, as they are afforded the power of deciding which students are "good" students and which ones are "poor" students. Consistent policy which places primary value on completion of work, attitude and behavior reinforces teachers' power. Students who fail to cooperate with their teachers (or who, in the words of one focus group member, do not "adapt to the teacher's style") are much less likely to do well in math than students who "give teachers what they want." Since mathematics plays a central role in a student's general high school education, his or her ability to exhibit desired behaviors will have serious consequences for his or her academic future.

Research Question #5: How is high school mathematics characterized by student resistance?

Investigation #1 relies upon regression to respond to research question 5. In responding to research question 1, path analysis supported an assumed causal relationship among grade level, track, degree of interaction in the math classroom, resistance to mathematics, mathematical confidence, and a student's math grade. The diagram best supported assumed causal relationships between resistance and interaction, and resistance and grade level. School mathematics is the site of tremendous student resistance, even in terms of students identified as "good" math students by their teachers. Students resist not only learning mathematical "content," they resist assimilation within the conforming, demoralizing, seemingly irrelevant, hostile environment of traditional mathematics classrooms.

Stepwise multiple regression with "resistance" as the dependent variable using all survey items as independent variables and replacing missing values with the mean produced the following equation (with $R = .716$): $y^{\wedge} = -.187(\text{confidence}) + .399(\text{somecan't}) - .219(\text{good}) - .181(\text{happy}) - .069(\text{academic grade}) + (.074)\text{tchrcall} - .058(\text{explain}) + .051(\text{smart}) - .052(\text{everyone}) + .048(\text{grade}) + .047(\text{quiet})$. According to this equation, the most significant predictors of student resistance to mathematics are accordance with the statement, "Some people can't learn to do math," confidence, and disagreement with the statements "I am good at math" and "I feel happy when I'm in my math class."

Teacher ratings of student math ability (collected in Investigation #2) also relate to student resistance to high school mathematics. **Teachers described "poor" math students using terms related to their classroom behavior or attitude.** Among these descriptors were "rude," "disrespectful," "not prepared for class," "lacking motivation," "refusing to go to the Homework Center," "having the maturity of a fifth-grader," "not caring," "talking all the time," "immature," "flaky," and "having no clue about school." One teacher commented that one of her "poor" students "actively keeps others from participation" as well. Each of these characteristics could be seen as a synonym for resistance, which equates to action in opposition to that expected by the teacher.

Students in focus groups (Investigation #3) spoke of choices they and others make with regard to their performance in math classes. One student explained how she brought her book home every night but never opened it. When asked why, she just shook her head and provided no explanation. One student suggested that "mathematics" should be stuff on worksheets. "You shouldn't have to use your book," he said. "You shouldn't have to read in order to get it done." Many students commented on the consequences of having a "bad" math teacher. Many students explained why people refuse to do homework. "Most of the time, homework just requires that students do the same thing over and over." "Some students have time pressures," "Some students don't have time to go through all the steps you need to go through to get the right answer to a problem." Tests cause resistance, many students complained that teachers count tests as too large a factor in a student's grade.

In summary, all focus group participants described some aspect of resistance to school mathematics. Only two of the "poor" math students named any specific mathematical evidence in support of their comments that school mathematics has nothing to do with their lives. These "poor" math students were able to explain their resistance to both the subject and to other factors (teachers, textbooks, systemic requirements), "average" and "good" students spoke generally about the reasons why they were resigned to "play the game" of school mathematics. Even when they stated they might "study engineering or science," affinity to mathematics never emerged as a reason. In short, successful players in the school math game view the subject as a necessary obstacle, at best a means to ends that appear vague and undefined.

Research Question #6: How does interaction/dialogue in the mathematics classroom affect student confidence in high school mathematics?

Interaction and dialogue are important factors in both student success and student failure in school math classrooms. They are important factors not so much for their "cognitive" merit, but more for their implications as the means for students to understand and gain control over their environments and others within these settings. Investigation #1 responds to research question 6 through path analysis and regression. The path diagram presented in response to research question #1 asserts a causal relationship among grade level, interaction, resistance to mathematics, student value of mathematics, objective view of mathematics, mathematical confidence, and academic grade in mathematics. The model supports a strong causal relationship between interaction and an objective view of mathematics. It also describes an indirect causal relationship between interaction and mathematical confidence. The largest path coefficient is that between interaction and student value of mathematics. The Pearson correlation between "interact" and "confidence" (significant at the .01 level) is .367. Pearson correlations of individual components of the composite variable "interact" with selected variables are shown in Table 22 below:

Table 22: Pearson Correlations between Key Variables

	confid	converse	tchrcall	explain	groups	resist	mthvalue
confid	1	.310*	.278*	.251*	.116*	-.559*	.728*
converse	.310*	1	.158*	.252*	.149*	.140*	.279*
tchrcall	.278*	.158*	1	.115*	.013	-.006	.200
explain	.251*	.252*	.115*	1	.064	-.179*	.270*
groups	.116*	.149*	.013	.064	1	-.013	.114*
resist	-.559*	-.140*	-.006	-.179*	-.013	1	-.352
mthvalue	.728*	.279*	.200*	.270*	.114*	-.352*	1

* $p < .001$

The composite variable "confidence" is most highly correlated with the composite variable "student's value of math" ($r = .728$). These composite variables have no contributing items in common. Table 34 also shows a strong negative correlation between "resistance" and "confidence." The strongest correlation between "confidence" and an interaction-related variable is $r = .310$ which describes the correlation between item #14 (I often hear conversations about math in my math class) and "confidence." Other significant correlations exist between "confidence" and item #19 (My math teacher calls on me often) ($r = .278$) and between "confidence" and item #15 (My math teacher asks me to explain my answers to math questions) ($r = .251$). Each of these correlations is significant at the .001 level. It is interesting to note the lack of correlation between items #21 (I often work with other students in groups to do math) and #15 (My math teacher asks me to explain my answers to math questions) and "resistance."

This last finding suggests that teachers who draw students into discussions or conversations through questions or other prompts may lower their students' resistance within the math classroom. Similarly, student resistance may be reduced through incorporation of collaborative learning opportunities into regular class activities. By

themselves, these findings have been supported by other research. What is troubling, however, is the resignation characteristic of students who exhibit less explicit resistance to mathematics. As Secada (1992) suggests, **focus on independent classroom strategies (like discussion or cooperative learning, for example) without considering their implications within a larger context is detrimental to students from subordinated populations. Using discussion and/or collaborative activities merely to lower student resistance without considering the critical consequences of this result would be and is worse than maintaining a traditional math class environment.**

Regression with "interact" as the dependent variable produced the following equation ($R = .492$) $y^{\wedge} = .215(\text{mthvalue}) + .157(\text{everyone}) + .072(\text{forward}) + .076(\text{memory}) + .059(\text{tchrperc}) + .042(\text{afraid}) + .039(\text{mathsci}) + .044(\text{happy}) - .048(\text{good}) + .037(\text{used}) - .048(\text{success})$. The most significant predictors of interaction in the math classroom, therefore, are the composite variable "math value" and responses to item #25 (My teacher believes that everyone can learn math). This suggests that classrooms in which students experience a significant amount of interaction are characterized by teachers who have high expectations and who encourage students to place a high value on an abstract conception of mathematics as well as the norms and cultural markers of traditional mathematics classrooms. That "memory" appears in the regression equation supports this idea since traditional mathematics instruction relies heavily on the value of algorithmic familiarity and applied repetition of skills students are expected to "master."

When asked to explain how they would describe the interaction typical of math classrooms, students in the focus groups for Investigation #2 talked about a variety of

factors. **Students agreed that teachers talked most often.** "The teacher takes 30 minutes to explain something. Then students get a chance to ask questions while they're working on their homework." A similar comment was, "[teachers] teach a lesson, give homework and move on, even if people are missing stuff." According to one student, "All students [in math classrooms] say the same things. That's how you can tell they're all equally smart." One student remarked that "it's distracting when people talk all the time" in math class, while another complained that "the teacher talks the most. Nobody else gets to say anything."

Students also made numerous comments regarding group work. "Groups are really good in math. They help with both studying and learning...students should have a choice about whether or not they want to work alone or in a group," said one student. "Groups improve the chances of people understanding." "[groups] hold people more accountable." "People in groups talk about math more. You have to work with people who will get the work done." "Sometimes groups are distracting. They take time from individual work." "I prefer to work with my friends outside of class. That way I won't have any outside conversations in the math class." These conflicting opinions about group work suggest that students experience group activities differently. Certainly this is a function of their teachers' purposes for- and pedagogical approaches to the use of group work as a classroom strategy. **Why and how groups are used are much more important issues than whether or not groups are employed in the first place.**

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Several students described the language they hear most often in math classes as well as the environment that is typical of math classrooms. "The atmosphere in math classrooms is very important. Sometimes students don't want to be there," said one student

Students tend to ask other students for help with problems rather than asking the teacher for help. [Students] use language like "put this number here, put that one there," etc. It's colloquial language. Students help each other to get the answer as fast as possible. They don't mess around with long explanations

One student observed that "teachers don't realize that math is another language." Another student suggested that teachers are sometimes "boring. Teachers need to be more active and less boring. Lectures put me to sleep. [Teachers] need to change the ratio of talk to explanation." One student said that "teachers should know the math, not just read it out of a book" and said that they should choose good examples. Finally, one student commented that "Some teachers don't know how to discipline their classes. They say there are consequences, but they're never enforced so students get away with anything." This suggests that teacher values and expectations are transmitted to students through the ways they use language in the classroom. Since teachers produce by far the most language in traditional math classrooms, students consider their language to be either positive or negative. Negative attributes include detachment from student concerns, problems and other issues students consider important. They also include lack of teaching or content-area "expertise."

Good teachers, on the other hand, "talk to students if they're having problems." They "work with you one on one and they get to know you." Good teachers "come to students, rather than expecting students to come to them." They "have a positive attitude" and are "in a good mood most of the time." Good teachers "give notes on the assignment," "help with homework," and "make sure students know how to do problems." They "make sure you know it" and they "explain." Finally, one student said that teachers "have to keep their students in check," meaning that they need to discipline the students in their classes. All students in the focus group seemed to agree with this statement. This implies that students prefer a caring teacher whose rules are explicit and consistent to a teacher who is unwilling or unable to clarify and enforce his/her expectations and standards.

Teachers who provided names for focus group members in Investigation #3 tended to include interaction-related comments when describing their "poor" math students rather than when describing their "good" math students. "Poor" math students are described as "talking all the time" (a comment repeated by three teachers), "having the maturity of a fifth-grader," "immature," or "wanting to be a clown." "Good" math students, on the other hand, tend to be described as quiet, resolved, hard-working, polite students. "Good" students "never make excuses," they are "quiet," "polite" (repeated three times) and they "pay attention." They have a "studious attitude" and they "try hard" (repeated twice). One teacher remarked that her "good" students "work together to solve problems" and "participate" (a comment repeated by another teacher). Another teacher

commented that her "good" students "ask lots of questions" (repeated twice) and are "engaged in class."

In summary, teachers seem to value student talk as long as it conforms to their expectations and supports activities directed by the teacher. Teachers disapprove of students who speak spontaneously, out of turn, or defiantly. These kinds of interaction with others cause teachers to label such students as "poor" math students, despite the absence of mathematical content associated with such comments.

Research Question #7: How does high school mathematics function to “objectify” students and teachers?

In Investigation #1, the Pearson correlation between item #22 (The way to succeed in math is to memorize things) and item #23 (Answers to math problems are either right or wrong) was .223 (significant at the .01 level). Based on data obtained from the validation of the survey (where the Pearson r between these two items was .78), the composite variable “object” was created and used for path analysis described in the response to research question 1. The path coefficient between the composite variable “interact” and the variable “object” was .209 and the path diagram does support assumption of a significant direct causal relationship between “object” and “student value of mathematics,” which in turn exhibited a strong direct effect on the composite variable “confidence.”

The variable “object” had significant (one-tailed, of .001) correlations with key variables. These include the composite variable “student value of mathematics” ($r = .235$), item #18 (I expect to use math in my life when I am an adult) ($r = .171$), item #25 (My math teacher believes that everybody can learn math) ($r = .175$), item #6 (Math is the most important subject in high school) ($r = .164$), item #4 (It is important to do well in math classes) ($r = .179$) and the composite variable “interact” ($r = .216$). Although these are not remarkably strong correlations, they do support the assertion that students’ value of mathematics is tied to their perception of the subject as a set of objective truths, a finite set of skills required to find the unique solution to standard problems, solutions which can be derived through application of certain algorithms.

Focus group members' comments in Investigation #2 provided interesting responses to the second research question. Besides providing a variety of responses to the question, "Why is mathematics required in high school?" students provided several comments that describe the way mathematics objectifies students and teachers. **Many students describe mathematics as a "thing" that is "thrown at" students and which usually "bounces off" of them.** One student explained that teachers "treat math like so many grains of sand that they try to keep from draining out the other side of your head once they put them in." Students describe ways in which they "do" math or in which they have math "done" to them. The topic of assessments in math evoked a number of comments and most students complained that teachers consider tests to be far too important when calculating grades. For example, one student complained that teachers often "change the way a problem is" and therefore the technique students know how to use become useless.

Most students described mathematics as "the steps you need to use to find the answer." When asked to define "mathematics," students provided answers ranging from "mind games" to "calculations" to "the study of different kinds of numbers." Answers to mathematical questions are assumed to exist and to be in the possession of the teacher. "Teachers always know the answers, but they forget that students don't know them." According to one student, people fail math because they "don't have time to go through all the steps you need to go through to get the right answer to a problem." Math is, according to one student, different from other academic subjects because "in order to use a square root, you have to learn about roots before you can use it, and once you know

what it is, you can only use the square root in math problems.” One student acknowledged that not all math classes are alike:

Tech Prep was useful. It dealt with how to relate math to situations. It was easier to understand. It wasn't a list of problems. It was situations. We had to do presentations when we were done. We had to learn not to be afraid, not to snap under pressure. We worked in groups to answer real questions.

School mathematics appears as a complex set of abstract and elusive “truths” that could be considered to resemble a kind of “scientific mythology” that is perpetuated without question from one generation to the next. Variations in the archetypal stories do indeed exist, but the general premise is that mathematics can or will explain and justify not only world phenomena but also its pre-eminence among academic subjects.

The results of factor analysis in Investigation #4 (transcript analysis) when responding to research question #3 suggest that a student's academic transcript represents one main factor that is composed of citizenship GPA, number of earned credits as a percentage of attempted credits, and academic GPA. Strong correlation between teacher rating of student math performance and overall GPA ($r = .708$ with $p < .000$) suggests that teacher expectations are closely tied to overall academic performance rather than to observation of a student's specialized talents in a particular field such as mathematics. In this sense, students and teachers are objectified through self-fulfilling expectations: students who function well in public school settings are more likely to be considered “good” math students than those who do not.

Chapter 5: Discussion

The most practical way to discuss the significance of this study's findings is to describe the ways in which the results inform understanding of mathematics education in a real, ongoing school setting. This discussion will be focused on the policy-related implications of responses to the research questions and seeks to provide insight into critically-informed *action* that might be suggested by responses to research questions located in a particular context. This context consists of a high school community located just north of the U.S./Mexico International Border where 70 percent of students are people of color. One particular aspect of mathematics policy forms the basis of the discussion. Over the past two years, members of Mar Vista's math department have focused their attention on the development of course objectives and their effects on student transition within the college-preparatory mathematics hierarchy of courses. Included in this discussion are issues related to alignment with California State "Standards" and negotiation required to allow community-generated objectives to function despite external criticism.

Mar Vista High School is home to approximately 1500 ethnically diverse students. Over two-thirds of students receive free- or reduced-price lunches. Every year for the past five years, between 10 and 12 percent of the graduating seniors go on to pursue higher educations at four-year colleges or universities. The graduating class of 1996 consisted of 220 students, over 500 students entered Mar Vista as freshmen four years

earlier. Fewer than ten percent of graduating seniors have met the University of California's "A-F" sequence of courses (which includes mathematics through trigonometry). Clearly, not all students at Mar Vista are prepared to enter and succeed in institutions of higher education.

The subject most students at Mar Vista fail is mathematics. In the 1996-97 school year, it had the lowest schoolwide subject-specific grade-point-average (1.6 on a 4.0 scale) and it is the greatest concern for counselors who are interested in preparing students for graduation. The class with the worst achievement in mathematics is the ninth grade. At the end of the first semester of the 1996-97 school year, over 40% of ninth-grade students had earned a "D" or an "F" in their math classes. Mathematics is the only department to offer courses on a semester-by-semester basis. Before Mar Vista changed its policy to allow every math course to be offered every semester, students who failed a course during the first semester lost an entire year of mathematics. Many students were therefore precluded from completing three years of college-preparatory mathematics during their four-year high school careers.

Data from other schools in the Sweetwater Union High School District resembles that from Mar Vista. According to one mentor teacher who is conducting research on students earning multiple "D's" and "F's," over 30% of all math grades given in the district do not qualify students to advance to subsequent mathematics courses. Since a student must earn a "C" or higher in order to go on, this means that almost a third of students do not proceed smoothly along a college-preparatory sequence of math courses.

The District has taken a number of steps to improve the situation. Among recent changes is adoption of an "Integrated Mathematics" series of courses called "Course 1," "Course 2" and "Course 3." These new courses integrate seven mathematical "strands" and replace the traditional Algebra-Geometry-Trig sequence of courses. According to preliminary results (the new materials have been used for two years), students who had previously failed only mathematics courses are 80% more likely to succeed in their math courses now that the Integrated Mathematics materials are being used. Students who failed mathematics and other subjects continue to do poorly.

Professional Development for Teachers

In July, 1996, Mar Vista High School received a \$300,000 grant from the California Academic Partnership Project (CAPP) which is funded by the state of California. The major objective of this three-year project was to increase the number of students admitted to four-year universities. Although this grant laid out a number of means to reach this end, one of its primary goals for 1997-98 is examination and development of mathematics standards. Accompanying this goal is the development and strengthening of support systems that have been found to be effective in improving student academic achievement. Additionally, the grant has a curricular focus. It therefore calls for development of materials and resources that help teachers improve their students' access to higher education.

Nine of Mar Vista High School's 12 mathematics teachers were selected to participate in a three-week-long Mathematics Leadership Institute (sponsored by the San Diego Mathematics Project). During this time, they were exposed to current research and theory dealing with mathematics education. The mornings were spent exploring mathematical investigations in a variety of areas ranging from statistics to calculus. The afternoons were devoted to discussion of policy issues. Evolving state mathematics standards were among the major topics discussed, as were equity issues.

When teachers returned to Mar Vista in late August, many voiced concerns about the new Integrated Mathematics materials, which were criticized for being "a mile wide and an inch deep." Teachers therefore decided to use two preservice staff development days to "weed" the materials and identify the objectives they considered to be most important for their students. To complete this process, they cross-referenced district course descriptions, drafts of the *California State Mathematics Standards*, input from the California Educational Round Table, the *California State Mathematics Frameworks*, and other documents. The challenge of producing a finite list of objectives acceptable to everybody was formidable.

From the beginning, teachers were urged to consider the impact of clear and achievable course objectives on student ability to transition successfully from one level of mathematics to another. In other words, they were asked to focus on ways in which students might more successfully "play" and "win" the "game" of high school mathematics. The general premise of the articulation meeting was that more students

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need to emerge at the end of their secondary math experience prepared to enter- and succeed in mathematics courses required by four-year colleges and universities.

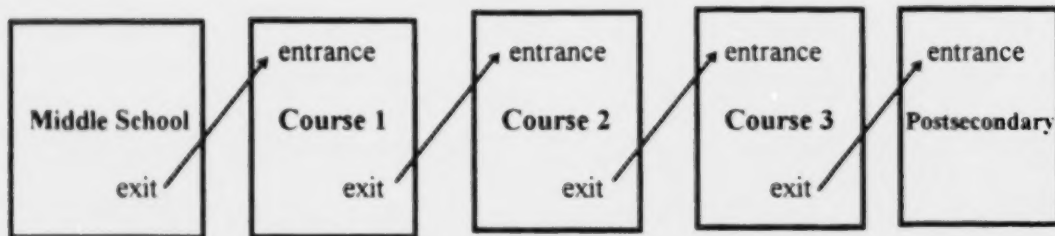
The adoption of an Integrated Mathematics curriculum opens college-preparatory mathematics to many more students than a traditional program and more explicitly differentiated tracks of mathematics study. It is therefore imperative that more students emerge from the pipeline ready to go on to college mathematics. Teachers were challenged to consider how they might align course requirements so that students might transition successfully along a continuum of several years. This perspective differs radically from the more common goal of students demonstrating "mastery" of discrete math skills at specific junctures in their school math careers before they are to be allowed to advance. If successful, we argued, our project would knit together the different levels of mathematics so that students would experience coherent passage from one clearly-defined set of objectives to another.

The project called for fundamental re-examination of beliefs when it sought to change the premise of the project from student deficits ("These kids just lack the bread-and-butter of algebra") to a more systemic view of how school mathematics functions as a *filter* rather than as a *pump* in the "machine" of educational opportunity. This challenge evoked much resistance in the form of defensive arguments ("I need some time to just get back to teaching instead of wasting it here") as well as expressions of fear ("I don't see how I can teach that... it's not in the book, you know"). In order to address teacher concerns and to allow participants to develop a genuine sense of agreement on project goals, we developed a series of workshops and secured funding to

allow teachers to meet off-campus for several days during the academic year. Our objective was to produce a finite list of objectives for each course, beginning with Course One. To facilitate this process, teachers engaged in a structured set of activities which all relied heavily on discussion and exchange of views.

Once teachers agreed on a final product consisting of limited and specific course objectives at each level, teachers met in groups by course level. Each group was asked to identify five "entrance expectations" (what they could reasonably expect all students to know or to be able to do) as well as five "exit outcomes" (what all students should reasonable know or be able to do upon earning a "C" in the given course). This first group discussion evoked debate and many important issues emerged. For example, teachers disagreed about which five concepts or skills were most critical to both student success in their course as well as likelihood of success in future math courses.

Once teachers developed these two sets of entrance/exit expectations, each group presented its lists to members of the other groups. This second stage of the process provided an important forum for inter-group discussion of key skills as well as alignment between the exit outcomes of one course and the entrance expectations of the next course. Teachers were asked to agree upon the relative order of the fifteen major objectives identified in this second exercise. What emerged from this collaborative, dialogical process formed the basis of future and more specific identification of course objectives. The model used as the basis for workshop activities is described in Figure 4 below.

Figure 4: Transition Through High School Mathematics

Merely reaching consensus on the fifteen entrance/exit objectives required that the math teachers reframe their previous conception of what constitutes a math class. At first, many teachers resisted the challenge of identifying five exit outcomes, claiming that “there’s no way we can just have five! I mean, Chapter Six has twelve just in itself!” Many teachers defined their current course content to be identical to the concepts and skills presented within the official, board-approved mathematics textbooks. They referred to the books’ tables of contents when identifying course objectives, listing them in the order in which they appeared in the textbooks. When faced with the restriction that they limit the number of exit outcomes to five, however, teachers began to pull away from their textbooks and to discuss the ways in which “mastery” of certain objectives would advance student success in future math courses. When presenting and discussing entrance and exit objectives as a whole group, the teachers surrendered ownership of favorite objectives (most notably factoring and conic sections) at one level so that these might more appropriately appear at different levels. Teachers left the initial discussions with not only an altered conception of their reasons for meeting in the first place, they also left with the

beginnings of a curricular "roadmap" which might assist them in planning everyday activities within their classrooms.

Following the initial entrance/exit objectives workshop, teachers met again in course-specific groups. This time they were tasked with developing a limited (less than 20) number of semester objectives that fit within the initial continuum of entrance- and exit objectives. This proved to be a very time-consuming process that also elicited much heated debate. Teachers introduced concerns about standardized tests like the SAT and the Golden State Exams. "We need to teach solid geometry earlier in the second semester or the kids will get rocked on the GSE," said one teacher. "Honors kids need to know about exponential growth and decay; it's always on the SAT," said another. Together they worked to examine the contents of the textbooks and other materials. The results of this process, which has been the ongoing focus of math department activities for the past two years, are provided as Appendix I.

Teachers not only use these documents to plan their semester curricula; they use time provided by grant-funded activities to review, revise, and re-evaluate various aspects of the documents, including relevant resources and support needed from peers. Many teachers cite objectives addressed by major exams using the exact language of the documents and new teachers are guided by more experienced teachers to plan their courses according to these documents. By any measure, all teachers in the math department agree that the documents, as well as the process that accompanies their refinement and application, have been very useful and instructive to them as teachers.

However, it is here that paradoxes begin to occur. Just as the process of standards development brought teachers together by a kind of shared vision for successful student transition within the math pipeline, reconciling such actions with an educational system hostile to many aspects of the project pitted teachers against each other. At the end of the fall semester of 1997 (the first semester after course-specific objectives were first developed), a box appeared in Mar Vista's main office. Within this box lay class sets of multiple-choice, coursewide final exams to be administered by every teacher. Concerns arose immediately: "Permutations are all over this exam! I thought we decided not to cover that until next semester," "Pascal's Triangle?! We decided to teach that with probability, right?"

After the initial reactions came concern about what action to take regarding the tests. The exams were accompanied by a directive from the District's Curriculum Office that all math teachers were to administer the tests, score them, and send results to the District Office, where they would be "analyzed." Fear unleashed itself. "They're going to use our scores for evaluations!" said one teacher. "We're all going to look like bad teachers since our students haven't seen so many of these items." "Should we have our students guess? Should we tell them to skip those questions? Can I just give them the right answers?" These concerns consumed much time at math department meetings. Finally, someone proposed that teachers identify the questions they considered "unfair," (i.e. concepts to which students had had zero exposure) and replace them with questions whose content was otherwise absent on the exam but present in course objectives. That

way, students' *scores* on the exams would more closely represent their "knowledge of mathematics "

Some teachers worked together to develop a set of questions for replacement purposes, however, some teachers abandoned the course objectives documents and "crammed" their students full of the concepts that they know would appear on the exam. Clearly, the collaboration, solidarity, and vision that characterized development of the course objectives documents was in danger of being overcome or reversed by pressure from the system outside of the school.

Discussions following this initial, "experimental" final exam experience centered upon the challenge of maintaining the integrity of our documents within the sometimes conflicting pressures of the system at large. We decided to show up en masse whenever the curriculum office sought teachers to help write future final exams; we came with our standards documents in our hands. Four of the six teachers at a recent work session were from Mar Vista. We gave up weekends so that we could "stack" the District Finals with our objectives. "If the test is going to contain a somewhat arbitrary collection of concepts," one teacher argued, "that collection might as well be Mar Vista's."

Math department inservice days were also spent reviewing ways in which our objectives documents aligned with the exams currently being used. Sometimes revision included merely adding emphasis to an idea. Other documents reflected more significant changes (such as the addition of probability to the first semester of Course Two) and the incorporation of ongoing research projects to be designed and directed by individual teachers in conjunction with their students. These "supplemental" research projects,

however, seldom aligned with the content of standardized tests, particularly the SAT-9 (Stanford Assessment Test) and STAR-9, its successor. Teachers who took the time to plan special, relevant lessons could be fairly certain that their students would see little, if anything, related to these projects on any standardized test. For students at Mar Vista, these tests include the districtwide final exam for each course, the Stanford Achievement Test (now the STAR test), the Golden State Exam, the PSAT, the SAT I and II, the ACT and perhaps Advanced Placement exams. These tests are used to rate the performance of both students and teachers in the district. Scores on the writing sample (which is required for graduation) are public knowledge; teachers whose scores are "noticeably low" receive computerized printouts of these scores along with handwritten notes from Principals or other authority figures. Teachers whose students earn low grades in their classes receive similar notes.

Our public education system (particularly the discipline of mathematics) has become frozen by these kinds of measures. Critical educators constantly seek to reveal the various ways in which these tests overtly and covertly strip marginalized students of power. Perhaps more important than that, however, is the critical educator's need to guide students through the minefield of testing intact. It is teachers' love for students that drives critical educators to both face the oppressive forces and to search for ways to overcome them. Central to this vision is critical engagement of both our colleagues and the system in which we function as teachers.

The schoolwide math GPA at Mar Vista High School at the end of the 1997-98 school year was 1.63 on a 4.00 scale. From a distanced perspective, it seems ironic that a

group of math teachers could feel they had made so much progress while their students continued to earn low grades. Most "logical" people in education would assume that a department of teachers who discussed and valued consistency in standards, expectations and course requirements would be able to "produce" students who succeeded in mathematics. Through a critical lens, however, the department's experiences are not at all surprising or counterintuitive. It is here that I would like to provide personal experiences gained as a classroom teacher.

What most of my colleague shudder to even consider is the way in which mathematics functions systemically to oppress students, particularly those who are members of subordinated populations. On one hand, teachers may be seen as powerful individuals who perpetuate a mythology that includes the preeminence of mathematics over all other subjects. This mythology derives its staying power from time-honored, seductive propaganda asserting that students will "use this stuff one day" when they are "out there, in the real world." On the other hand, teachers may serve as "institutional agents" who help students mediate the world around them. This "world" includes much more than quadratic equations and trinomials; it includes awareness of many implicit codes and norms that students much adopt if they are to function successfully within advanced math courses and higher education in general.

Student ability to apprehend these codes and norms may seem to necessitate students' abandonment of their individual and cultural backgrounds and ideologies. That this must occur for students to gain access to four year universities is the dominant message in many math classes, where a student's behavior is often perceived to be a more

significant indicator of mathematical promise than his or her ideas, questions, or thinking. In order for students to avoid being completely subordinated by the expectations of the culture of power (or traditional culture of higher education and its prerequisites), students must learn to mediate the world around them. Especially in fields like mathematics, where so many oppressive forces are allowed to operate unchallenged, students need opportunities to talk with each other and to form meaning from their various experiences. The role of dialogue in the mathematics class has been relegated to marginal notes associated with the "mathematical communication" strand described by the *California Mathematics Framework* (1990). A critical teacher relies upon dialogue in his or her classroom not just because it is considered "effective" as an instructional strategy to be used in "delivering" the math curriculum to one's students. Dialogue is an important means for helping both students and teachers to develop critical consciousness. "Founding itself upon love, humility, and faith, dialogue becomes a horizontal relationship of which mutual trust between the dialoguers is the logical consequence" (Freire, p. 72).

Merely encouraging students to "talk" in class is not the same as providing them with the conditions that allow for real dialogical engagement of important issues and ideas. Likewise, although the teacher leads discussion in the classroom, he or she must guard against adopting traditional control of the agenda and terms for dialogue within a classroom. "posing reality as a problem does not mean sloganizing it means critical analysis of a problematic reality" (Freire, p. 149). This is no easy task, especially in today's era when teachers face increased criticism (from sources both within and outside

of schools) related to the “achievement of student outcomes” and the “mastery” of certain fixed objectives.

This summer I taught Integrated Math Course 2 (the second in my district’s three-course, college-preparatory sequence) to students who were all required to retake the course in order to move forward. To say that these students dreaded mathematics is understatement. Several students told me that the only reason they were taking the class was to “get a credit” they needed for graduation from high school. Only a very few of my students indicated that they had any plans whatsoever to continue study of mathematics after they left high school. Although I tried to create introductory journal questions that dealt with events in the world around them (like World Cup soccer math tied to the Pythagorean Theorem and coordinate geometry), many of my students remained apathetic at best, often creating “discipline problems” by refusing to bring their textbooks, refusing to participate, or failing to complete required assignments. My students didn’t want to talk about anything unless they were convinced it had nothing at all to do with math. When I managed to link their comments to applications of mathematical ideas, they groaned and clammed up.

I became more and more frustrated as I contemplated my choices: either allow students to “get the credit” they needed merely because they were cooperating and donating their summers “to the cause” (as inane as it seemed, since they didn’t indicate that they had any plans to attend college in the first place), or fail all the students who were unable to “produce” evidence that they had acquired skills and knowledge that would predict success in Course 3. Of course, these “skills and knowledge” had much

more to do with compliance and willingness to conform to their teacher's expectations than they did with "mathematical" predictors of success.

About halfway through the summer, the *San Diego Union Tribune* published the results of the 1998 Stanford Achievement Test (a norm-referenced test administered to all students at the end of each academic year). The results were reported as percentiles without any explanation of what this kind of data actually represents. Along with the percentile scores for each school in every local district, the newspaper had included the percentage of "LEP" (Limited English Proficient) students at each school. At the beginning of class one day, a student made some comment about how, yet again, Mar Vista appeared at the bottom of the list of High Schools. I had cut the article out of the newspaper and I decided to abandon the "plan" for the day (I think I was in the middle of trying to teach students how to solve quadratic equations) and we began to discuss the issues associated with the irresponsibly incomplete newspaper article. "What does this number (percentile) here represent? Does anybody know the difference between a *raw score* and a *percentile rank*?" I asked. This question provoked student questions like "Is that test fair to kids who don't even speak English?" and "Who writes questions for that test? Who scores them? Who pays for it all?"

When we started looking at the scores of other schools in the area, students revealed the kind of self-image the media has forced them to absorb. I asked, "What school do you think has the highest scores?" They correctly nominated the school serving students from the highest socio-economic backgrounds. Then I asked them to compare the scores of students at this school to the number of "LEP" students reported. They

noted an inverse relationship that basically applied to data from each school. "Where do you think San Ysidro Middle School ranks?" I asked. Students cast disparaging comments and concluded that San Ysidro kids must be really "dumb." Many of those very students sitting in the classroom lived in San Ysidro! They had just classified themselves as inferior, stupid losers.

Then we again compared percentile ranks to "LEP" populations. "What do you think most people will conclude after seeing these two numbers (the percentile ranks and the LEP numbers) together?" I asked. Many students made comments about how schools populated by immigrant students or students who spoke languages other than English at home would be perceived as "bad" schools and that the opposite would be true for schools serving more privileged (and English-dominant) students. Then students again mentioned the fact that the test was administered only in English and that students who couldn't speak this language obviously wouldn't be able to do well, regardless of the test's content or the students' knowledge. One student suggested that the newspaper's presentation of the data lay blame for poor scores on just those students most powerless to positively affect the performance of the aggregate student body.

It was during this conversation that many of my students first displayed genuine interest in any class discussion. Many students who were otherwise blatantly oppositional proved to be attentive listeners as their peers asked questions and expressed frustration at how they were treated in school. The discussion lasted for more than an hour, quadratic equations languished on overhead transparencies near an abandoned overhead projector. What spoke most loudly to me as a teacher was what happened to the class once we

returned to the quadratics after finishing the discussion. I asked students to spend several minutes writing notes in their journal about how they might provide a rebuttal to the article in the form of editorial letters. Then, when I asked them to graph a quadratic function on the same page, every student set himself to the task. It was the first time that the whole class was engaged in an activity that could be identified as "pure mathematics."

Of course, that's exactly what it wasn't. My students must have decided that they would take a chance on learning a discrete mathematical concept because they had gained a hint of awareness of the ways in which some people use math to relegate others to subordinate positions in society and keep them there. Perhaps they learned that I really did want them to "get through" the summer's math adventure without sacrificing either their souls or their opportunity to advance to higher levels. Perhaps, as Freire writes, dialogue cannot exist without humility (on behalf of me as a teacher); clearly I had decided that the discussion was more important than my preset agenda for the day. Although members of my class continued to display resistance after that lesson, we moved forward more as a team. "Nor yet does dialogue exist without hope" (Freire p. 72).

Other Key Aspects

I recognized that if I were to bring a critical approach to mathematics education I would have to engage the various levels on which traditional ideology and hegemony function to maintain the status quo. In light of this observation, it is important to describe ways in which participants at various levels in the educational system were co-opted into

participating in Mar Vista's attempts to revise its math curriculum, thereby entering a critical dialogue. Two important aspects of Mar Vista's these discussions were ongoing participation by representatives of postsecondary institutions and the accepted departmental philosophy that students should be enrolled in college-preparatory mathematics unless one or more of a limited number of extenuating circumstances applies to a student.

One person who was perhaps most openly optimistic about Mar Vista's course objectives development work was a UCSD professor who plays several roles of systemic importance. First, he directs San Diego's Mathematics Diagnostics and Testing Program (MDTP), a state-funded project whose goal is to develop, administer and evaluate standard criterion-referenced multiple-choice exams for mathematics courses at a variety of levels. Although MDTP exams are not intended to be used as tools for student placement in math courses, they are frequently misused by schools as "weeding" devices, tests students must "pass" in order to advance.

Second, the professor serves as a member of Mar Vista's California Academic Partnership Project (CAPP) Grant Planning Committee and is a member of the Imperial Beach community. His comments regarding Mar Vista's course objectives were considered valuable because they linked concerns for Mar Vista's student transition to that world of college mathematics we hopes students would be able to enter. In short, successful student transition is worth very little if students are unable to survive in college mathematics.

Another aspect has complex implications within a critical framework. On one hand, Mar Vista's policy of enrolling all students in college-preparatory mathematics suggests that all students have access to an education that will provide numerous opportunities. Indeed, in 1998, Mar Vista ranked second in the Sweetwater Union High School District among all high schools in terms of the percentage of student applications which met University of California subject area admission requirements. Although Mar Vista has the smallest senior class in the district, 60% of those students who applied to the U.C. system were determined to be "qualified" to succeed there (in terms of the courses they successfully completed while in high school) if admitted. One of the largest schools in the district with comparable socio-economic status reported that 32% of its applicants met the requirements.

Since mathematics is the subject most responsible for incomplete U.C. admission requirements, Mar Vista's comparatively large percentage of "qualified" applicants suggests that its inclusive college-prep policy is preparing students to succeed in the future. On the other hand, however, resistance to mathematics led over 120 of Mar Vista's 1998 graduating class of 250 students not to take mathematics during the second semester of their senior year.

If, as results of the study suggest, teachers consider students enrolled in college-preparatory mathematics to be generally "better" math students than those not on this track, then it follows that as many students as possible ought to be "spared" the "non-college-prep" label. However, if resistance to traditional mathematics is most acute at the most advanced levels of school mathematics, and if many seniors opt out of study at these

levels, then mandatory enrollment within this "college-prep" system is not so clearly desirable or beneficial to students. Since mathematics course-taking in secondary school is a key indicator of student success in institutions of higher education, this issue deserves attention.

One of the most important consequences of Mar Vista's attention to course objectives and standards has been much increased recognition that higher expectations for student success in school mathematics must be accompanied by more responsive student support systems and programs. Of course, the purposes served by these programs from traditional and from critical perspectives seldom appear alike.

The California Academic Partnership Project (CAPP) at Mar Vista High School is based on the principle that standards documents must be developed in conjunction with the support systems which can improve student success. Among these supportive programs is an after-school Tutoring and Homework Center which is open every day from 2.30 p.m. until 8.00 p.m. During this time, at least one math teacher, two college tutors and four high school calculus students are available to assist Mar Vista High School students with their homework and projects. The Center has proven to be very popular, with over 100 students visiting per day. Other supportive programs involve trips to universities for shadowing of students and library research, development of language-conscious mathematics lessons, strengthening of the AVID (Advancement Via Individual Determination) site team on campus, and support for a full-time mathematics resource teacher who helps teachers provide students with access to computers, software and graphing calculators.

Significant Connections to the Literature

Apple (1995) and Fine (1995) observe that mathematics reform has failed to address the *depth* of current problems. This is certainly true in the case of one school's experiences. While teachers and students have made progress in improving student achievement as measured by standardized measures, few are willing to question what it is that "high school mathematics" has come to mean. On the day before summer school begins, mathematics teachers are scurrying around the school getting their first week's worth of worksheets Xeroxed. One might ask, "how can they possibly know what they need to teach their students before their students have even arrived?" A more insightful question would be, "how did the school system arrive at a place where teachers must contend with over 45 students in two-hour-long class periods?" An alternative question might be, "why do teachers understand their role to be transmitters of decontextualized, abstract information that their students have already failed to absorb? Why will the 'mathematics' taught to this summer's repeating students be the same as the mathematics the students failed to 'master' during the previous year?" Clearly, Secada's (1995) concept of mathematics education as a matrix of often conflicting factors is important.

Superficially, it seems ironic that teachers who have otherwise proved to be so progressive as advocates for their students are at the same time purveyors of the most traditional mathematics curriculum and instructional techniques. Less superficially, these (albeit progressive) teachers are trapped in a web of bureaucratic constraints and

traditional ideologies. As Frankenstein (1987) and Freire (1970) note, the most dominated people have the most fragmented localized awareness of their situation and therefore are unable to think dialectically about it. Teachers in a classroom with 47 students who have returned from mathematical failure to face the monster yet again can hardly be expected to seek the "reflective" knowledge in which mathematics is taken to be a human activity.

Results of this study point to at least several major factors that should be located in the "matrix" defining problems facing the mathematics education community. Among the significant factors contributing to a student's resistance to mathematics and his/her value of mathematics are his/her grade (9-12) and whether or not he/she is enrolled in a college-preparatory mathematics course. Similarly, mathematical confidence is influenced by these preceding variables and exerts an effect on students' academic grades. Although it is not surprising to note that mathematical confidence seems to have a strong causal relationship with a student's academic grade, it is more interesting to note the connection between mathematical confidence and students' value of mathematics. It is also interesting to ask why a student's enrollment in a college-prep math course and the degree of interaction he/she experiences in his/her math class share a significant correlation with mathematical confidence.

Again, the degree of interaction a student experiences in his/her math class seems to have a causal relationship with his/her value of mathematics. One might ask why interaction in math class contributes to higher value of the subject. Why does the degree of interaction seem to have a causal effect on mathematical confidence? From a critical

perspective, students who engage in meaningful dialogue in their math classrooms may be more "empowered" than those who do not. Perhaps such interaction allows students to challenge the positivistic nature of traditional school mathematics, which is often considered neutral, objective, value-free and outside of human consciousness. Perhaps such interaction allows students to link the two sides of the often "clear" dichotomy between "school math" and "real-world" math. Perhaps this makes possible an understanding of mathematics as a set of "dialogical" ways of knowing (Skovsmose, 1993). Perhaps such interaction allows teachers to talk "to" students rather than "at" students. Perhaps such interaction reduces the inherent inclination of traditional mathematics pedagogy to sort students by mathematical "ability" since more students have a chance to show that they know something about the subject. Of course, it is important to ask whether or not greater amounts of student interaction in math classrooms take place in collaborative or cooperative settings, where students have an opportunity to work together to solve problems or to complete assignments.

Having made these observations, it is important to note that summer school classrooms must accommodate at least 45 desks. Given the dimensions of the desks (which require students to face one direction and give them almost no leeway to move in any direction) and the classrooms (which are rectangular, equipped with blackboards and an overhead projector), teachers are virtually precluded from conducting significant collaborative or discussion-based lessons until well into the summer when 1) several students have dropped the course, 2) students and teachers develop a working relationship of respect which adheres to the classroom rule of one person talking at a time, or 3) the

teacher takes members of the class outside of the room in groups and leaves the rest working on projects or other activities unsupervised. A more likely scenario is that teachers are forced to employ the "foreign method" (Tate, 1995), which creates cultural discontinuity between the teacher and the students and among students themselves. The teacher most likely relies on worksheets and other approaches typical of "banking" models of instruction. Above all else, the teacher is concerned that students "demonstrate" the "mastery" of the subject that is required for progression to the next course in the mathematics sequence. Teachers who question traditional *reasons* for teaching mathematics most certainly are not present in summer school, where critical alternatives to traditional mathematics education almost elude the imagination.

Returning to an earlier observation, students at different grade levels seem to experience mathematics differently. More specifically, students in the early years of high school express less resistance to mathematics than do students in the later years. Younger students do not feel as "afraid" in their math classes; they do not feel as "helpless." It is interesting to note that students in 12th grade seem to find little or no connection between "school math" and "real-world" math; 9th-graders, on the other hand, have a less negative reaction. It is especially interesting to compare this result with the fact that ninth-graders express hope that they may one day use math in their adult lives, juniors are the least hopeful about this possibility, followed by seniors. Students in 9th grade express greater value of mathematics, seniors express the lowest value of mathematics. Finally, one must note that younger students seem to believe that everybody has the potential to learn

mathematics much more than older students do. What happens to students over the course of their experiences with high school mathematics to produce these results?

Before discussing the theoretical explanations for these phenomena, it is helpful to examine another set of contrasts which emerged as results from the study. Students in lower levels of mathematics (not necessarily based on students' grades 9-12) appear to have a different perception of mathematics and mathematics education than do their peers in more advanced levels of mathematics. The most marked difference exists between students enrolled in Math B and students in Calculus or PreCalculus. Students in Math B have a much lower concept of their abilities as math students. They do not feel that they are as "good" at math, they feel more helpless in their math classes, they have lower academic grades, and they do not have as clear an understanding of what is required to earn an A. Not surprisingly, therefore, they seem to have significantly higher levels of resistance to mathematics and they indicate that they plan to become mathematicians or scientists much less than do their Calculus/PreCalculus-enrolled peers.

Comparisons between Math B and Calculus/Precalculus are interesting and suggestive because students in these classes are mostly the same age. Math B is the non-college-prep successor to Math A, which in the past was called everything from "pre-algebra" to "consumer math." The only way a student can advance to Math B under current math department guidelines is if he or she earns a grade of "D" in Math A. Students who fail Math A must repeat Math A until they earn a grade other than "F." Students who earn a "C" or better are promoted to Course 1, the first in the college-prep sequence of courses. Students in Math B, therefore, have probably experienced

significantly negative experiences in high school mathematics courses. Students in PreCalculus or Calculus, on the other hand, have advanced along the mathematics course continuum from Course 1 to Course 2 to Course 3 to PreCalculus and, ultimately, to Calculus. These students have succeeded in earning grades that allowed them to progress to advanced levels of mathematics, it is unlikely that a student will reach PreCalculus having failed one or more math courses.

This set of comparisons suggests that students in college-preparatory mathematics courses experience mathematics differently than students in non-college-prep courses. The results of this study, like many other studies, support this assertion. Most significantly, students in the non-college-prep track seem to consider themselves significantly less "good" at math than their college-prep peers. They have lower academic grades and they seem to believe that they "can't do math" as well as their college-prep peers. Non-college-prep students say they hate math more than do college-prep math students and non-college-prep students say they know what is required to earn an A less well than do college-prep students. This last comparison evokes another general observation: college-prep students seem to know how to "play the mathematics education game" better than non-college-prep students. This is manifest in other results as well.

"Playing the game" is a theme that emerges repeatedly in both the results of this study and in the related literature. Mathematics education is the ultimate example of the "meritocracy" that seems to govern public schools: this system functions to enforce the hegemony of mathematics as primary among academic subjects as well as the tacit understanding that college-preparatory students are the ones who will inherit the academic

(and therefore economic and political) throne of the future. The mathematics curriculum is designed to create an "expertocracy" that serves the "selective tradition" described by Apple (1990) and others. One focus group member commented that "successful" math students "adapt to the teacher's style." While all students seemed to agree that students with "A's" in mathematics do not necessarily know more about mathematics than students without "A's," they all agreed that "successful" math students "do their work." This comment was repeated by numerous math teachers, who proposed that the most important characteristic of a "good" math student was his or her willingness to "do the assigned work."

Seymour (1995) observed that one of the biggest obstacles to women seeking to major in mathematics and science is the nature of the academic majors themselves. Perhaps the same can be said of mathematics programs in high schools. Student failure may have much less to do with the subject itself than the system in which students are expected to "perform." How, one might ask, is a student to "perform" as one member of a 47-student class guided by traditional administrative expectations and assessed via a district-wide, multiple-choice exam? Surely the "system" in which students continue to fail deserves critique. It enforces social stratification in school and beyond; it serves as a magnet for student resistance. As evidenced by the huge number of students who must take summer school because they failed mathematics courses during the year, student "failure" in the current mathematics education system is tremendously expensive. A cynical analyst might observe that unless students failed math courses during the year, math teachers (and the administrative system built around remediation) would not have

employment during the summer. What, then, is the real incentive to empower all one's students to obtain passing grades the first time through a math course?

It would be inadequate and irresponsible to leave teacher characteristics out of a final discussion of critical mathematics education versus traditional models. Clearly, the ethnic, linguistic, and gender diversity that characterizes the Mar Vista High School mathematics department has been a significant factor in its progressive actions. An outside observer might ask, then, why people with such diverse life experiences (three of whom - bilingual - graduated from Mar Vista High School itself before obtaining teaching credentials) have not further challenged so many aspects of the mathematics education system. One example is the significant difference between teacher ratings (as good, average or poor math students) of students from San Ysidro versus teacher ratings of students from Imperial Beach. Additionally, students who had not passed the AB65 English language writing sample were much less likely to be named "good" math students than those who had passed the sample. This is surprising since half the math teachers are themselves bilingual, native speakers of a language other than English. Why would home address and English language proficiency correlate so strongly with teacher rating of students' mathematics abilities?

Perhaps the answer to the question posed above is related to the ways in which mathematics serves as a "technology" in society, a force with "formatting" (Fischer, 1993) power which functions as both a means and as a system. The answer may also be related to the inherent psychological bias of mathematics education, where Piagetian concepts of development are applied to processes of mathematical learning. Most simply, the answer

may be related to the fact that in order to become a mathematics teacher in the public school system, one must "win" at "the game" of mathematics education. Nobody who refuses to "play" will be allowed into a classroom. To be a critical educator and to be allowed to function freely in a system characterized by traditional values and expectations are, indeed, two goals that seem to contradict each other.

Thus we arrive at the final conclusion of the study: the tensions created by attempting to move along the continuum between traditional mathematics education and critical mathematics education are essential for greater empowerment of students and teachers who seek more fully human and equitable ways of living and learning. The other key component to successful transition along this continuum is ongoing, honest, reflective dialogue that is accompanied by theory and action. If the idea of statistical analysis applied to questions of student resistance seems aberrant, it creates tension. If multiple investigations generate a matrix of factors contributing to one's understanding of mathematics education, tension is produced by efforts to reconcile conflicting results. Dialogue must address these tensions. It must call into question more than what has previously been under the magnifying glass and, in this case, address the most fundamental questions: Why do we require mathematics education for all students in high school? What is the essential nature of the mathematics that we "teach?" What, besides mathematics, do students learn from being in mathematics classrooms? Each of these questions could, and should, be addressed from a critical perspective.

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Mathematics Education Survey: Thank You for Participating!

What math class are you taking this semester? (circle one): math 7 math 8 crs 1 crs 1 HN other none

What grade are you in? (circle one): 7 8 Are you male or female? (circle one): M F

What scholarship grade do you usually get in your math classes? (circle one): A B C D F

For each statement below, please circle the number that best reflects your opinion:

	Agree ←-----→ Disagree				
1. I understand why I have to take math in school.	5	4	3	2	1
2. I am good at math.	5	4	3	2	1
3. I hate math.	5	4	3	2	1
4. It is important to do well in math classes.	5	4	3	2	1
5. I was good at math in elementary school.	5	4	3	2	1
6. Math is the most important subject in high school.	5	4	3	2	1
7. People who do well in math will succeed in life.	5	4	3	2	1
8. I use math from school to solve problems in my daily life.	5	4	3	2	1
9. I am expected to stay quiet in my math class.	5	4	3	2	1
10. I look forward to working on math problems.	5	4	3	2	1
11. Math is my favorite subject.	5	4	3	2	1
12. My math course is challenging enough for me.	5	4	3	2	1
13. People who do well in math are smart.	5	4	3	2	1
14. I often hear conversations about math in my math class.	5	4	3	2	1
15. My teacher asks me to explain my answers to math questions.	5	4	3	2	1
16. I feel angry when I'm in my math class.	5	4	3	2	1
17. My math teacher thinks I am a good student.	5	4	3	2	1
18. I expect to use math in my life when I am an adult.	5	4	3	2	1
19. My math teacher wants me to succeed in math.	5	4	3	2	1
20. I can't do math.	5	4	3	2	1
21. I often work with other students in groups to do math.	5	4	3	2	1
22. The way to succeed in math is to memorize things.	5	4	3	2	1
23. Answers to math problems are either right or wrong.	5	4	3	2	1
24. I feel helpless in my math class.	5	4	3	2	1
25. My teacher believes that everybody can learn math.	5	4	3	2	1
26. I know what is required to earn an A in math.	5	4	3	2	1
27. Some people can't learn to do math.	5	4	3	2	1
28. I have enough opportunities to get help and support at school.	5	4	3	2	1
29. I feel afraid when I'm in my math class.	5	4	3	2	1
30. I will become a mathematician or a scientist	5	4	3	2	1

Encuesta de matemáticas: ¡Gracias!

¿Cuál clase de matemáticas estas tomando este semestre? (circula uno): math 7 math 8 crs I crs IIB otra nada

¿En que grado estás? (circula uno): 7 8 Genero: (circula uno): M F

¿Qué calificación obtienes normalmente en tus clases de matemáticas? (circula uno): A B C D F

Por cada oración circula el numero que mejor refleja tu opinion: De acuerdo ← → No estoy de acuerdo

	5	4	3	2	1
1. Entiendo por que necesito tomar matemáticas.	5	4	3	2	1
2. Me considero buen(a) estudiante en matemáticas.	5	4	3	2	1
3. No me gustan las matemáticas.	5	4	3	2	1
4. Es importante sacar buenas calificaciones en matemáticas.	5	4	3	2	1
5. Era bueno(a) en matemáticas en primaria.	5	4	3	2	1
6. La clase más importante en la preparatoria es matemáticas.	5	4	3	2	1
7. Los que hacen bien en matemáticas tendran éxito en su vida.	5	4	3	2	1
8. Uso matemáticas que aprendí en la escuela para resolver problemas en mi vida diaria.	5	4	3	2	1
9. Se espera que este callado en mi clase de matemáticas.	5	4	3	2	1
10. Me gusta trabajar en problemas de matemáticas.	5	4	3	2	1
11. Mi clase favorita es matemáticas.	5	4	3	2	1
12. Mi clase de matemáticas esta suficiente estimulante.	5	4	3	2	1
13. Los que hacen bien en matemáticas son inteligentes.	5	4	3	2	1
14. A menudo escucho conversaciones de matemáticas.	5	4	3	2	1
15. Mi maestro(a) me hace explicar mis repuestas a preguntas de math.	5	4	3	2	1
16. Me siento enojado(a) en mi clase de matemáticas.	5	4	3	2	1
17. Mi maestro(a) piensa que soy buena estudiante.	5	4	3	2	1
18. Especto usar matemáticas en mi vida cuando sea adulto.	5	4	3	2	1
19. Mi maestro(a) de matemáticas me quiere tener éxito en matemáticas.	5	4	3	2	1
20. No puedo hacer matemáticas.	5	4	3	2	1
21. A menudo me junto en un grupo para trabajar en mis matemáticas.	5	4	3	2	1
22. La forma de tener éxito en la clase de matemáticas es memorizar.	5	4	3	2	1
23. Respuestas a preguntas de matemáticas estan correctas o incorrectas.	5	4	3	2	1
24. Me siento perdido(a) en mi clase de matemáticas.	5	4	3	2	1
25. Mi maestro(a) piensa que todos podemos aprender matemáticas.	5	4	3	2	1
26. Entiendo lo que necesito de hacer para poder obtener una "A" en mi clase de matemáticas.	5	4	3	2	1
27. Algunas personas no pueden aprender matemáticas.	5	4	3	2	1
28. Tengo suficiente oportunidad para recibir ayuda en la escuela MVM.	5	4	3	2	1
29. Tengo miedo cuando estoy en mi clase de matemáticas.	5	4	3	2	1
30. Voy a hacer un(a) matemático(a) o un(a) científico(a) en mi vida.	5	4	3	2	1

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Please return this completed form to
K. Czajkowski before 2/27/98. Thanks!

(teacher name)

Two students I consider “good” math students are

(student name)

in

(math course)

and

(student name)

រំព

(math course)

These students are “good” math students because:

Two students I consider “average” math students are

(student name)

in

(math course)

and

(student name)

10

(math course)

These students are “average” math students because:

Two students I consider "poor" math students are

(student name)

in

(math course)

and

(student name)

in

(math course)

These students are "poor" math students because:

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Course One - Major Topics for Semester one

		SUHSD Standard #
Topic/Objective	Key Book Section(s)	Crs. Outline Reference(s)
Use "Guess and Check" throughout the semester to solve equations, in addition to other methods.	CPM Mtls and other Suppl. Mtls	
1. Use the distributive property to simplify expressions and equations and to combine like terms.	1-5, 2-6, 5-2	I-2
2. Use order of operations to evaluate integer and variable expressions.	1-4	I-2
3. Perform operations with positive and negative numbers.	2-2	I-2
4. Evaluate base 10 powers of variables and perform operations using rules of exponents.	1-3, 2-3	I-1
5. Classify angles (vertical, straight, right, acute, supplementary, etc.) and triangles (acute, obtuse, scalene, isosceles, right, etc.)	2-5,	II-6, II-7
6. Emphasize: a) Solve equations with one variable. b) Rewrite formulas & solve for one variable in terms of another. c) Model situations with equations and solve.	2-7, 2-8, and all of Ch. 5	V-1, III-1
7. Find mean, median, mode, range & outliers. Display & analyze survey-generated data with frequency tables and histograms.	3-2, 3-4	IV-1
8. Solve and graph inequalities on a number line. Model situations and graph with one variable.	3-3, 5-4,	I-5
9. Graph functions using tables, including TI calculators.	4-6, 4-7	III-11
10. a) Find area of polygons. b) Find the circumference and area of circles	5-7 Suppl. Mtls.	II-1

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Course One - Major Topics for Semester Two

Topic/Objective	Key Book Section(s)	SUHS Standard #
		Crs. Outline Reference(s)
1. a. Use ratio and proportion to solve problems. b. Find the length of an arc and the area of a sector of a circle.	6-1, 6-3 6-5 7-3, 7-6	I-6, III-6
2. a. Find the slope and intercepts of a line from its graph or its equation. b. Use slope/intercept(s) to graph a line. c. Model direct variation to make predictions.	7-2, 7-4 8-1, 8-3	III-3, III-7 III-13
3. Demonstrate tangent as an application of slope and use the tangent ratio to solve problems.	7-1	II-5, III-12
4. Graph inequalities on the coordinate plane.	8-6	III-4
Emphasize: 5. a. Write equations of lines in slope-intercept form and standard form. b. Model real-world situations with two variables (control, dependent).	7-4, 8-1 8-2, 8-3	III-2, III-8 V-4
6. Perform operations with radicals, find square roots & cube roots; solve equations using square and cube roots.	9-2, 2-9	I-3, I-4
Emphasize: 7. Use the Pythagorean theorem in problem-solving.	9-1 Suppl.	II-8
8. a. Use the distributive property (FOIL) to expand binomials. b. Factor polynomials using GCF. c. Factor trinomials (with a=1 only)	10-5, 10-6 Supplement	III-11

Throughout both semesters, engage in mathematical communication using investigations, projects, oral presentation, and portfolios.

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Course Two
Major Topics: Semester One

Topic/Objective	Key Book Section(s)	Crs. Outline Reference(s)
1. Recognize the relationship between the domain and range of functions.	2.1	3(1)
2. Demonstrate the ability to use slope and y-intercept or two points to write the equation of a line.	2.2	
3. Find the Surface Area and Volume of prisms, cones, cylinders, spheres.	2.4, supplements	
4. Distinguish between the graphs of linear and nonlinear functions.	2.5, 9.4	
5. Simplify expressions with positive, fractional, zero, and negative exponents.	2.6, 9.2, supplements	1(1)
6. Solve systems of equations using the most appropriate strategy (to include graphing, substitution, and elimination).	3.1, 3.2, 3.4, supplements	3(3), 5(4)
7. Identify critical points for graphs of quadratic equations.	4.1	3(6), 3(7)
8. Compare families of quadratic equations.	4.2, SDAIE	3(9)
9. Solve quadratic equations using the zero product property, quadratic formula and/or graphing and identify critical points.	4.1, 4.3, 4.4	3(8), 3(6), 3(7)
10. Factor quadratic expressions.	4.4, supplements	3(8)
11. Find the probability of single, independent and dependent events.	6.1, 6.2	
12. Differentiate between permutations and combinations and apply the appropriate formulas to solve problems.	6.3, 6.4	

Notes: Doubling/halving should be addressed in supplemental SAT workshops.
Honors sections: explore complex numbers.

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Course Two
Major Topics: Semester Two

Topic/Objective	Key Book Section(s)	Course Outline Reference(s)
1. Recognize and classify quadrilaterals and describe their properties (especially regarding diagonals).	5.1, supplements	6(6)
2. Use the midpoint formula to find the midpoint or endpoint of a given segment.	5.3	2(2)
3. Transform geometric figures through translation, dilation and rotation.	5.4	2(6)
4. Label coordinates of figures in prime position.	5.5	2(5), 2(7)
5. Use Pascal's Triangle or the binomial theorem to expand binomials.	6.6, 6.9 (parts)	1(3), 4(1)
6. Differentiate between inductive and deductive reasoning.	1.5, 1.6	5(2)
7. Differentiate between valid and invalid arguments.	7.3	5(1)
8. Identify the hypothesis and conclusion of an "if-then" statement, write converses of statements and provide counterexamples.	7.2, 7.4	5(3)
9. Graph the solution to a system of inequalities.	7.4	3(10)
10. Demonstrate the ability to create 2-column algebraic proofs.	(7.5), 7.6, 7.8, 8.1	6(8)
11. Use postulates, theorems, definitions and rules of logic to find measures of angles formed by transversals and parallel lines.	7.7, 7.8, 8.1	2(12), 6(7)
12. Use properties of triangles and quadrilaterals (especially angle sum theorems and special properties of isosceles and equilateral triangles) to solve for unknown values.	8.2, 8.6	
13. Apply rules of similarity and congruence to triangles.	8.3, 8.4, 8.5	2(8), 2(9)
14. Use the geometric mean and other relationships to solve proportions.	8.7	
15. Use properties of 45-45-90 and 30-60-90 degree triangles to find side lengths and angle measures.	8.8	2(9), 2(11)
16. Define and use sine, cosine and tangent to find side lengths and angle measures in triangles.	8.8	2(10)

Notes: Introduce constructions *prior* to proofs (Olinger has materials).

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Course Three - Major Topics for Semester one

Topic/Objective	SUHSD Standard #	
	Key Book Section(s)	Crs. Outline Reference(s)
1. Solve a system of equations using multiple methods, including matrices on a graphic calculator, multiplication/elimination, and substitution. (Emphasize 2x2 systems and use of Gr. Calc. for 3x3 systems)	1-4, 1-5	I-2, III-11
2. a. Recognize functions, domains and ranges. b. Use linear, piecewise, abs. value, quadratic, and rational functions to model real life problems. c. Find the composite of two functions.	Ch 2 (Do all sections.)	III-1 to III-3
3. Evaluate formulas and other expressions using operations such as opposite, reciprocal, absolute value, raising to a power, taking a root, grouping symbols and order of operations in solution of applied problems.	Ch. 2	I-1, I-3
4. Write 2-column, and coordinate proofs.	3-1 to 3-8 (Skip sec. 3-3 & 3-9.)	II-1
5. Use properties of circles, polygons and angles to solve problems. Include: Sums of interior & exterior angles in polygons, circumscribed polygons/inscribed circles, relationships between inscribed angles and arcs of circles, tangents and secants of a circle.	3-4 to 3-6	II-3
6. a. Describe and continue patterns. b. Write and use explicit and recursive formulas for sequences. c. Identify arithmetic & geometric sequences and write equations for them. d. Find the sums of finite arithmetic series, and finite and infinite geometric series.	4-1, 4-2 4-3 4-4 4-5 to 4-7	III-4 to III-6

Recommended topics covered in earlier courses which should be reviewed for SAT tests: Pythagorean theorem, Pythagorean triples, special right triangles, proportionality & similarity, area formulas, circumference & perimeter, three-dimensional geometry.

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Course Three - Major Topics for Semester Two

Topic/Objective	SUHSD Standard #	
	Key Book Section(s)	Crs. Outline Reference(s)
1. a. Use exponential functions in problem solving. b. Find and graph inverse relations. c. Use properties of logarithms to evaluate logarithmic functions; use logarithmic functions to solve problems.	5-1 to 5-8	III-7 to III-9
2. a. Identify, from either equations or graphs, the conic sections. b. Graph and name the appropriate parts of each conic section. c. Put equations into standard conic form by completing the square.	Ch. 9 from old Merrill Algebra 3/4 book	
3. a. Use polar coordinates to locate points and make conversions between polar and rectangular coordinates. b. Use algebra and graphing to solve vector problems.	8-1, 8-2 8-3, 8-4	
4. Graph and solve parametric equations.	8-5	III-10
5. Use the law of cosines and the law of sines to find measures of sides and angles in triangles.	8-6, 8-7	II-4
6. a. Graph periodic functions, extending definitions of sine, cosine and tangent to angles greater than 90 and less than zero. b. Recognize period, amplitude, horizontal & vertical translation and stretch, and reflection of a periodic function. c. Use the unit circle to find trig. ratios	10-1 to 10-5 Suppl. Mtl.	II-4
If time permits at end of semester: 7. a. Identify types of frequency distributions and use them to draw conclusions. b. Use mean, range and standard deviation to compare data sets, and recognize properties of normal distributions.	6-1 6-2, 6-3	IV-1 & IV-2 IV-3
If time permits at end of semester: 8. a. Use simulations and geometric models to solve problems. b. Find compound probabilities c. Recognize independent and dependent events, and find conditional probabilities.	7-1, 7-3 7-2 7-4	IV-4 to IV-7

Recommended topics covered in earlier courses which should be reviewed for SAT tests: Pythagorean theorem, Pythagorean triples, special right triangles, proportionality & similarity, area formulas, circumference & perimeter, three-dimensional geometry.

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Mathematics Education Survey: Thank You for Participating!

What math class are you taking this semester? (circle one): math 7 math 8 crs 1 crs 1 HN other none

What grade are you in? (circle one): 7 8 Are you male or female? (circle one): M F

What scholarship grade do you usually get in your math classes? (circle one): A B C D F

For each statement below, please circle the number that best reflects your opinion:

Agree ←————→ Disagree

1. I understand why I have to take math in school.	5	4	3	2	1
2. I am good at math.	5	4	3	2	1
3. I hate math.	5	4	3	2	1
4. It is important to do well in math classes.	5	4	3	2	1
5. I was good at math in elementary school.	5	4	3	2	1
6. Math is the most important subject in high school.	5	4	3	2	1
7. People who do well in math will succeed in life.	5	4	3	2	1
8. I use math from school to solve problems in my daily life.	5	4	3	2	1
9. I am expected to stay quiet in my math class.	5	4	3	2	1
10. I look forward to working on math problems.	5	4	3	2	1
11. Math is my favorite subject.	5	4	3	2	1
12. My math course is challenging enough for me.	5	4	3	2	1
13. People who do well in math are smart.	5	4	3	2	1
14. I often hear conversations about math in my math class.	5	4	3	2	1
15. My teacher asks me to explain my answers to math questions.	5	4	3	2	1
16. I feel angry when I'm in my math class.	5	4	3	2	1
17. My math teacher thinks I am a good student.	5	4	3	2	1
18. I expect to use math in my life when I am an adult.	5	4	3	2	1
19. My math teacher wants me to succeed in math.	5	4	3	2	1
20. I can't do math.	5	4	3	2	1
21. I often work with other students in groups to do math.	5	4	3	2	1
22. The way to succeed in math is to memorize things.	5	4	3	2	1
23. Answers to math problems are either right or wrong.	5	4	3	2	1
24. I feel helpless in my math class.	5	4	3	2	1
25. My teacher believes that everybody can learn math.	5	4	3	2	1
26. I know what is required to earn an A in math.	5	4	3	2	1
27. Some people can't learn to do math.	5	4	3	2	1
28. I have enough opportunities to get help and support at school.	5	4	3	2	1
29. I feel afraid when I'm in my math class.	5	4	3	2	1
30. I will become a mathematician or a scientist.	5	4	3	2	1

Encuesta de matemáticas: ¡Gracias!

¿Cuál clase de matemáticas estás tomando este semestre? (circula uno): math 7 math 8 crs 1 crs 11IN otra nada

¿En que grado estás? (circula uno): 7 8 Genero: (circula uno): M F

¿Qué calificación obtienes normalmente en tus clases de matemáticas? (circula uno): A B C D E

Por cada oración circula el numero que mejor refleja tu opinion: De acuerdo ← → No estoy de acuerdo

1. Entiendo por que necesito tomar matemáticas.	5	4	3	2	1
2. Me considero buen(a) estudiante en matemáticas.	5	4	3	2	1
3. No me gustan las matemáticas.	5	4	3	2	1
4. Es importante sacar buenas calificaciones en matemáticas.	5	4	3	2	1
5. Era bueno(a) en matemáticas en primaria.	5	4	3	2	1
6. La clase más importante en la preparatoria es matemáticas.	5	4	3	2	1
7. Los que hacen bien en matemáticas tendran exito en su vida.	5	4	3	2	1
8. Uso matemáticas que aprendí en la escuela para resolver problemas en mi vida diaria.	5	4	3	2	1
9. Se espera que este callado en mi clase de matemáticas.	5	4	3	2	1
10. Me gusta trabajar en problemas de matemáticas.	5	4	3	2	1
11. Mi clase favorita es matemáticas.	5	4	3	2	1
12. Mi clase de matematicas esta suficiente estimulante.	5	4	3	2	1
13. Los que hacen bien en matemáticas son inteligentes.	5	4	3	2	1
14. A menudo escucho conversaciones de matemáticas.	5	4	3	2	1
15. Mi maestro(a) me hace explicar mis repuestas a preguntas de math.	5	4	3	2	1
16. Me siento enojado(a) en mi clase de matemáticas.	5	4	3	2	1
17. Mi maestro(a) piensa que soy buena estudiante.	5	4	3	2	1
18. Especto usar matemáticas en mi vida cuando sea adulto.	5	4	3	2	1
19. Mi maestro(a) de matemáticas me quiere tener exito en matematicas.	5	4	3	2	1
20. No puedo hacer matemáticas.	5	4	3	2	1
21. A menudo me junto en un grupo para trabajar en mis matemáticas.	5	4	3	2	1
22. La forma de tener exito en la clase de matemáticas es memorizar.	5	4	3	2	1
23. Respuestas a preguntas de matemáticas estan correctas o incorrectas.	5	4	3	2	1
24. Me siento perdido(a) en mi clase de matemáticas.	5	4	3	2	1
25. Mi maestro(a) piensa que todos podemos aprender matemáticas.	5	4	3	2	1
26. Entiendo lo que necesito de hacer para poder obtener una "A" en mi clase de matemáticas.	5	4	3	2	1
27. Algunas personas no pueden aprender matemáticas.	5	4	3	2	1
28. Tengo suficiente oportunidad para recibir ayuda en la escuela MVM.	5	4	3	2	1
29. Tengo miedo cuando estoy en mi clase de matemáticas.	5	4	3	2	1
30. Voy a hacer un(a) matemático(a) o un(a) científico(a) en mi vida.	5	4	3	2	1

(teacher name)

Two students I consider **"good"** math students are

(student name) in _____
(math course)
and

(student name) in _____
(math course)

These students are **"good"** math students because:

Two students I consider **"average"** math students are

(student name) in _____
(math course)
and

(student name) in _____
(math course)

These students are **"average"** math students because:

Two students I consider **"poor"** math students are

(student name) in _____
(math course)
and

(student name) in _____
(math course)

These students are **"poor"** math students because:

Course One - Major Topics for Semester Two

SUHS
Standard #

Topic/Objective	Key Book Section(s)	Crs. Outline Reference(s)
1. a. Use ratio and proportion to solve problems. b. Find the length of an arc and the area of a sector of a circle.	6-1, 6-3 6-5 7-3, 7-6	I-6, III-6
2. a. Find the slope and intercepts of a line from its graph or its equation. b. Use slope/intercept(s) to graph a line. c. Model direct variation to make predictions.	7-2, 7-4 8-1, 8-3	III-3, III-7 III-13
3. Demonstrate tangent as an application of slope and use the tangent ratio to solve problems.	7-1	II-5, III-12
4. Graph inequalities on the coordinate plane.	8-6	III-4
Emphasize: 5. a. Write equations of lines in slope-intercept form and standard form. b. Model real-world situations with two variables (control, dependent).	7-4, 8-1 8-2, 8-3	III-2, III-8 V-4
6. Perform operations with radicals, find square roots & cube roots; solve equations using square and cube roots.	9-2, 2-9	I-3, I-4
Emphasize: 7. Use the Pythagorean theorem in problem-solving.	9-1 Suppl.	II-8
8. a. Use the distributive property (FOIL) to expand binomials. b. Factor polynomials using GCF. c. Factor trinomials (with $a=1$ only)	10-5, 10-6 Supplement	III-11

Throughout both semesters, engage in **mathematical communication** using investigations, projects, oral presentation, and portfolios.

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Course Two

Major Topics: Semester One

Topic/Objective	Key Book Section(s)	Crs. Outline Reference(s)
1. Recognize the relationship between the domain and range of functions.	2.1	3(1)
2. Demonstrate the ability to use slope and y-intercept or two points to write the equation of a line.	2.2	
3. Find the Surface Area and Volume of prisms, cones, cylinders, spheres.	2.4, supplements	
4. Distinguish between the graphs of linear and nonlinear functions.	2.5, 9.4	
5. Simplify expressions with positive, fractional, zero, and negative exponents.	2.6, 9.2, supplements	1(1)
6. Solve systems of equations using the most appropriate strategy (to include graphing, substitution, and elimination).	3.1, 3.2, 3.4, supplements	3(3), 5(4)
7. Identify critical points for graphs of quadratic equations.	4.1	3(6), 3(7)
8. Compare families of quadratic equations.	4.2, SDAIE	3(9)
9. Solve quadratic equations using the zero product property, quadratic formula and/or graphing and identify critical points.	4.1, 4.3, 4.4	3(8), 3(6), 3(7)
10. Factor quadratic expressions.	4.4, supplements	3(8)
11. Find the probability of single, independent and dependent events.	6.1, 6.2	
12. Differentiate between permutations and combinations and apply the appropriate formulas to solve problems.	6.3, 6.4	

Notes: Doubling/halving should be addressed in supplemental SAT workshops.
Honors sections: explore complex numbers.

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Course Two

Major Topics: Semester Two

Topic/Objective	Key Book Section(s)	Course Outline Reference(s)
1. Recognize and classify quadrilaterals and describe their properties (especially regarding diagonals).	5.1, supplements	6(6)
2. Use the midpoint formula to find the midpoint or endpoint of a given segment.	5.3	2(2)
3. Transform geometric figures through translation, dilation and rotation.	5.4	2(6)
4. Label coordinates of figures in prime position.	5.5	2(5), 2(7)
5. Use Pascal's Triangle or the binomial theorem to expand binomials.	6.6, 6.9 (parts)	1(3), 4(1)
6. Differentiate between inductive and deductive reasoning.	1.5, 1.6	5(2)
7. Differentiate between valid and invalid arguments.	7.3	5(1)
8. Identify the hypothesis and conclusion of an "if-then" statement, write converses of statements and provide counterexamples.	7.2, 7.4	5(3)
9. Graph the solution to a system of inequalities.	7.4	3(10)
10. Demonstrate the ability to create 2-column algebraic proofs.	(7.5), 7.6, 7.8, 8.1	6(8)
11. Use postulates, theorems, definitions and rules of logic to find measures of angles formed by transversals and parallel lines.	7.7, 7.8, 8.1	2(12), 6(7)
12. Use properties of triangles and quadrilaterals (especially angle sum theorems and special properties of isosceles and equilateral triangles) to solve for unknown values.	8.2, 8.6	
13. Apply rules of similarity and congruence to triangles.	8.3, 8.4, 8.5	2(8), 2(9)
14. Use the geometric mean and other relationships to solve proportions.	8.7	
15. Use properties of 45-45-90 and 30-60-90 degree triangles to find side lengths and angle measures.	8.8	2(9), 2(11)
16. Define and use sine, cosine and tangent to find side lengths and angle measures in triangles.	8.8	2(10)

Notes: Introduce constructions *prior to* proofs (Olinger has materials).

Course Three - Major Topics for Semester one

Topic/Objective	Key Book Section(s)	SUHSD Standard # Crs. Outline Reference(s)
1. Solve a system of equations using multiple methods, including matrices on a graphic calculator, multiplication/elimination, and substitution. (Emphasize 2x2 systems and use of Gr. Calc. for 3x3 systems)	1-4, 1-5	I-2, III-11
2. a. Recognize functions, domains and ranges. b. Use linear, piecewise, abs. value, quadratic, and rational functions to model real life problems. c. Find the composite of two functions.	Ch 2 (Do all sections.)	III-1 to III-3
3. Evaluate formulas and other expressions using operations such as opposite, reciprocal, absolute value, raising to a power, taking a root, grouping symbols and order of operations in solution of applied problems.	Ch. 2	I-1, I-3
4. Write 2-column, and coordinate proofs.	3-1 to 3-8 (Skip sec. 3-3 & 3-9.)	II-1
5. Use properties of circles, polygons and angles to solve problems. Include: Sums of interior & exterior angles in polygons, circumscribed polygons/inscribed circles, relationships between inscribed angles and arcs of circles, tangents and secants of a circle.	3-4 to 3-6	II-3
6. a. Describe and continue patterns. b. Write and use explicit and recursive formulas for sequences. c. Identify arithmetic & geometric sequences and write equations for them. d. Find the sums of finite arithmetic series, and finite and infinite geometric series.	4-1, 4-2 4-3 4-4 4-5 to 4-7	III-4 to III-6

Recommended topics covered in earlier courses which should be reviewed for SAT tests: Pythagorean theorem, Pythagorean triples, special right triangles, proportionality & similarity, area formulas, circumference & perimeter, three-dimensional geometry.

ev. May 1998

Course Three - Major Topics for Semester Two

Topic/Objective	SUHSD Standard #	
	Key Book Section(s)	Crs. Outline Reference(s)
1. a. Use exponential functions in problem solving. b. Find and graph inverse relations. c. Use properties of logarithms to evaluate logarithmic functions; use logarithmic functions to solve problems.	5-1 to 5-8	III-7 to III-9
2. a. Identify, from either equations or graphs, the conic sections. b. Graph and name the appropriate parts of each conic section. c. Put equations into standard conic form by completing the square.	Ch. 9 from old Merrill Algebra 3/4 book	
3. a. Use polar coordinates to locate points and make conversions between polar and rectangular coordinates. b. Use algebra and graphing to solve vector problems.	8-1, 8-2 8-3, 8-4	
4. Graph and solve parametric equations.	8-5	III-10
5. Use the law of cosines and the law of sines to find measures of sides and angles in triangles.	8-6, 8-7	II-4
6. a. Graph periodic functions, extending definitions of sine, cosine and tangent to angles greater than 90 and less than zero. b. Recognize period, amplitude, horizontal & vertical translation and stretch, and reflection of a periodic function. c. Use the unit circle to find triq. ratios	10-1 to 10-5 Suppl. Mtl.	II-4
If time permits at end of semester:		
7. a. Identify types of frequency distributions and use them to draw conclusions. b. Use mean, range and standard deviation to compare data sets, and recognize properties of normal distributions.	6-1 6-2, 6-3	IV-1 & IV-2 IV-3
If time permits at end of semester:		
8. a. Use simulations and geometric models to solve problems. b. Find compound probabilities c. Recognize independent and dependent events, and find conditional probabilities.	7-1, 7-3 7-2 7-4	IV-4 to IV-7

Recommended topics covered in earlier courses which should be reviewed for SAT tests: Pythagorean theorem, Pythagorean triples, special right triangles, proportionality & similarity, area formulas, circumference & perimeter, three-dimensional geometry.

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